Comparison of Autoregressive Moving Average and State Space Methods for Mean Monthly Time Series Modelling of River Flows in Labrador and South East Quebec

By

Carissa Sparkes, P.Eng.

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Abstract

Time series data such as monthly stream flows can be modelled using time series methods and then used to simulate or forecast flows for short term planning. Short term forecasting can be applied, for example, in the hydroelectric industry to help manage reservoir levels at storage dam facilities as well as energy supply throughout the year at run of river facilities. In this study, two methods of time series modelling were reviewed and compared, the Box Jenkins autoregressive moving average (ARMA) method and the State- Space Time-Series (SSTS) method. ARMA has been used in hydrology to model and simulate flows with good results and is widely accepted for this purpose. SSTS modelling is a method that was developed in the 1990s for modelling economic time series and is relatively unused for modelling streamflow time series. The work described herein focuses on modelling stream flows from three selected basins in Labrador and South-East Quebec, the Alexis, Ugjoktok and Romaine Rivers, using these two time series modelling methods and comparing results. The ARMA and SSTS models for each study basin were compared for fit of model, accuracy of prediction, ease of use and simplicity of model to determine the preferred time series methodology approach for modelling the flows in these rivers. The performance of the methods for both the model fit as well as the forecast accuracy was measured using Nash Sutcliffe Coefficient, Median Absolute Percentage Error and Mean Squared Deviation error calculations. It was concluded that the SSTS method for these three rivers produced better fitting models than the ARMA method, but was generally equivalent in prediction accuracy to the ARMA method. The ARMA models, in contrast, were easier to diagnose and could be used to produce flow simulations, which was challenging using SSTS.

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List of Symbols, Nomenclature or Abbreviations

ACF - Autocorrelation Function

AIC – Akaike Information Criterion

ARIMA - Autoregressive Moving Average with Differencing

ARMA - Autoregressive Moving Average

LCL - Lower Confidence Limit

LOWESS - Locally Weighted Scatterplot Smoothing

MAPE - Median Absolute Percentage Error

MSD - Mean Squared Deviation

NSE - Nash Sutcliffe Efficiency Coefficient

PACF - Partial Autocorrelation Function

PAR - Periodic Autoregressive

RJ - Ryan-Joiner normality test

SARIMA - Seasonal Autoregressive Moving Average with Differencing

SDF - Spectral Density Function

SSTS - State-space time-series

UCL - Upper Confidence Limit

Glossary

Akaike Information Criterion- A criterion used to measure the quality of a model representation of data compared to the actual data. The smaller the AIC value, the better the model represents the actual data.

Autocorrelation - Linear dependence of a time series variable with itself over two points in time. In the case of this thesis, the time series variables are monthly streamflows

Autocorrelation Function - Measures the similarity between observations in a time series as a function of the time between the observations

Deterministic - In state space time series analysis, the component baseline does not change over the time period

Frequency - The time interval between repeating observations of a time series

Homoscedasticity - When the variance of a time series remains constant over time

Lag - Distance in time between observations in a time series

Level - The intercept in a classical regression. For state space models, the level can change over time

LOWESS - Method for using locally weighted linear regression to smooth data

MAPE - A method for measuring accuracy of a model, whereby the percentage of error is calculated between each actual value and the estimated (modelled) value. The MAPE value is the median of the absolute values of the percentage error for each data point.

MSD - A method for measuring accuracy of a model, whereby the average of the square of the deviations between the actual value and the estimated (modelled) values is calculated

NSE - A method for measuring accuracy of a model, whereby the residual variance and data variance are used to calculate fit.

Partial autocorrelation function - Measures the autocorrelation between observations as a function of time between the observations, with the linear dependence removed

Principle of Parsimony- When there is a choice between two competing hypotheses, other things being equal, the simplest explanation is preferred

Residual - The difference between the predicted value and the actual measured value

Seasonality - A characteristic of a time series where the data changes predictably and repeats over the calendar year. The recurrence of these characteristics is typically associated with a season

Spectral density function - The process of decomposing a periodic time series into combinations of sinusoids.

Stationarity - Statistical properties of a function do not change over time. In a time series, the mean of the series remains constant over time

Stochastic - In state-space time series analysis, the baseline varies over the time period

Time Series State - The process related to the evolution of observations measured over time

Trend - Overall change in the mean of a time series over time

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Chapter 1 Introduction

1.1 Background

Developing or managing river systems is important in the field of water resources. Stream flow analysis is used to determine if flows are sufficient and reliable for a project. For developing a run-of-river hydroelectric project, for example, the engineer uses monthly stream flow analysis to determine whether a stream can meet the energy demand throughout all months of the year. In addition, short term forecasting can be used to help manage water at dam and hydroelectric facilities. Part of this design process includes developing a stream flow model based on historical flow records, simulating flows from the model to determine whether the model provides a good representation of the historical flows and then using the model for forecasting and simulation.

For the purposes of this study, the historical flow records used are mean monthly flows recorded at hydrometric stations. The data is arranged in chronological order and is known as a time series. Mean monthly flows will often display a seasonal component such that flows will be higher during periods of higher rainfall, runoff and/or snow melt. This seasonal component is important to capture in a stream flow model since, in the case of hydroelectric projects, determining whether monthly energy demand can be met all months of the year is essential to the success of such a project.

For the purposes of this research, it was important to select rivers that have natural rather than regulated flows, so that the natural characteristics of the rivers, like seasonality, were modelled. Many rivers in Labrador and South-East Quebec are undisturbed by development that would affect runoff and flow characteristics and as a result, this region was suitable as the general study area.

The primary industry accepted method for developing seasonal hydrologic models for time series data is mainly based on Box Jenkins methodology. Another method for developing seasonal models, which is not typically used for hydrologic modelling of time series data, is the State-Space Time Series (SSTS) method developed by Harvey (Harvey, 1989; Commandeur and Koopman, 2007). The SSTS method has primarily been used for economic time series model development and has rarely been used for hydrological modelling. The Box Jenkins ARMA method (Box et al, 2008) is well known and has been used in modelling of hydrometric time series since the early 1970s. This method was selected for comparison with the SSTS method for modelling stream flows in this study, with primary interest being in accuracy of the model to reproduce and predict the actual flow values for a number of rivers in the region.

1.2 Objectives

The goal of this thesis is twofold: to compare a well-known hydrological modelling method; namely the Box Jenkins ARMA method, to a less well known hydrological method, namely the SSTS method, and to determine if there were similarities in the selected models of different sized rivers across basins with differing characteristics. Each study river had both an ARMA and an SSTS model developed. The results of the models were compared to determine which method produced the best fitting model compared to the actual data set and which also most closely forecasted a reserved subset from the actual data set. A comparison was made between the models using Nash Sutcliffe Efficiency Coefficient (NSE), Median Absolute Percentage Error (MAPE) and Mean Squared Deviation (MSD) to determine fit as well as forecast accuracies for the models.

1.3 Outline of Thesis

This thesis analyzes and models monthly flows of selected rivers in Labrador and South East Quebec. Chapter 1 includes an introduction to the area of study as well as the objectives regarding modelling monthly time series flow. Chapter 2 provides information regarding data collection and data screening to facilitate the selection of the final rivers used for modelling and analysis. Two methodologies for modelling the selected rivers, Autoregressive moving average and state space time series, are presented in Chapter 3. Analytical results for each of the three rivers using both methodologies, as well as forecasting and longer term simulation capabilities presented in Chapter 4. Chapter 5 provides a comparison of both modelling methods and Chapter 6 includes conclusions of the analyses and recommendations for possible application of each methodology.

Chapter 2 Selection of Streamflow Series for Modeling

The criteria used for the selection of the streamflow series were natural flow rather than regulated, longest length of record possible and continuous record minimizing periods with missing data. These criteria were used to select streamflow records from rivers in the study area for detailed analysis.

2.1 Study Area

The area of study was defined as Labrador and South-East Quebec, along the portion of Quebec between Labrador and the Gulf of St. Lawrence. This area was selected due to the pristine nature of many of the basins as well as the number of hydrometric gauges in the area. Figure 2-1 depicts the locations of 79 hydrometric stations in Labrador and South-East Quebec. The length of record at these stations ranges between 8 and 60 years, some stations being on regulated rivers and some not.

The study area was divided into two subareas: North and South of the approximate latitude of Goose Bay. Geologic and surface conditions in general appear to be different between areas north and south of Goose Bay: the ground is rocky with less storage ability north of Goose Bay compared to south of Goose Bay. As runoff can be affected by vegetation type and density as well as ground infiltration and storage, the approximate latitude of Goose Bay was chosen as the divide separating the north region from the south region. All data were taken from databases of Environment Canada (2013) and the Province of Quebec (2013).

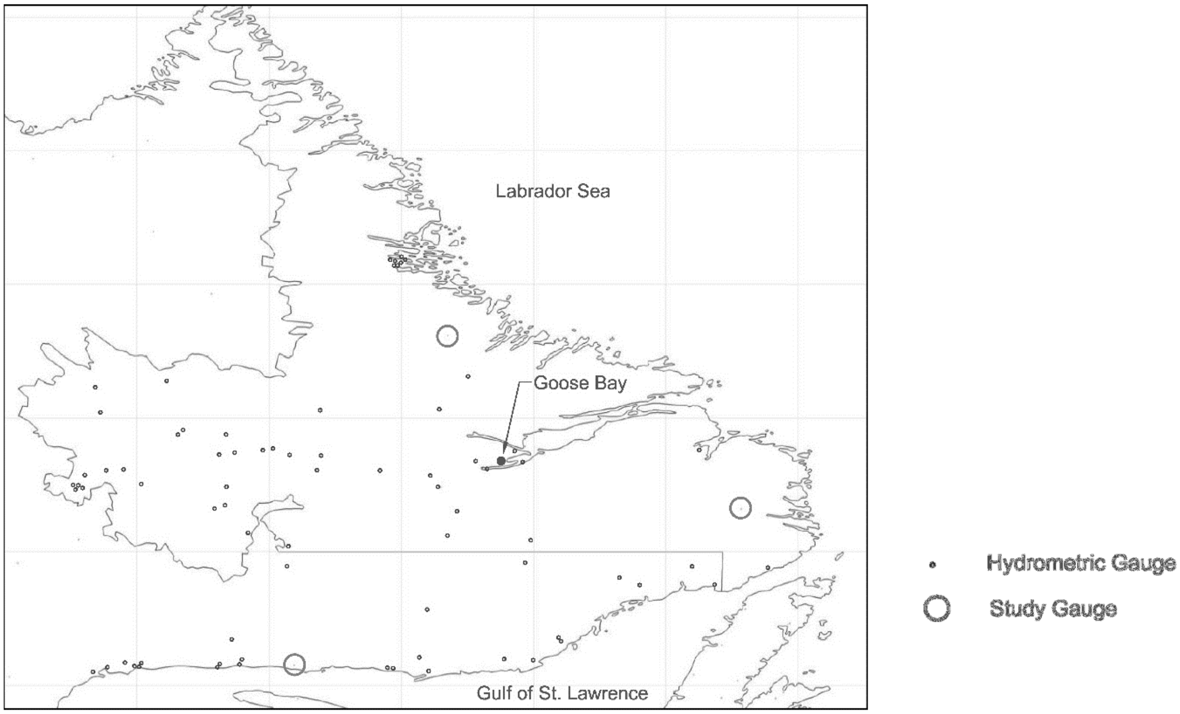


Figure 2-1: Hydrometric Stations in Study Area (Environment Canada, 2013)

2.2 Data Screening

For the purposes of this study, rivers were examined for length of record, regulation, and data continuity. Gauges with more than 15 years of data, on non-regulated rivers and having as much continuous data as possible, or as little missing data as possible, remained on the list for further examination.

In addition to these initial screening criteria, the preference was to eliminate all but three rivers, with the remaining each having different basin sizes upstream of each gauge, basin aspects and geographic locations. As a result of varying ground conditions, an effort was made to select rivers from both north and south regions. In addition to geographic location, drainage basin size as well as drainage aspect were also taken into account. This variation in the selected study rivers was preferred for the comparison of the two methodologies as well as to determine if there were similarities in the models across the three different river locations.

Having records from 79 gauge locations seemed promising, however the records from most of the hydrometric stations did not meet the screening criteria.

Table 2-1 includes the 33 stations having more than 15 years of data. Of these stations, those highlighted did not meet the remaining initial screening criteria, including being regulated and having discontinuous data, and were removed from the study. Of the 79 original records, only 12 remained on the list. These were further reviewed based on geographic location, drainage aspect and drainage size. In addition, in order to study rivers with a varied cross section of physical features and characteristics, at least one river from each of the northern and southern regions was selected, as well as at least one river draining to each of the Gulf of St. Lawrence and the Labrador Sea.

Table 2-1: Hydrometric Stations with More than 15 Years of Record in Labrador and South-East Quebec

Gauge Name	Drainage Area km^2	Years	Regulated	Continuous	Missing Data	State	North or South
Aganus River	5590	21	no	yes	none	discontinued	south
Alexis River near Port Hope Simpson	2310	34	no	yes	none	active	south
Ashuanipi River at Menihek	19000	59	yes			discontinued	north
Atikonak Lake at Gabbro lake	21400	38	yes			discontinued	
Atikonak river above Panchia lake	15100	25	no	no	1983 -98	active	south
Big Pond Brook below Big Pond	71.4	17	no	no	6 months	active	north
Churchill River above Upper Muskrat	92500	60	yes			active	south
Churchill River at Flour Lake	33900	17	yes			discontinued	north
Churchill River at foot of Lower Muskrat		16	yes			discontinued	south
Churchill River at powerhouse	69200	39	yes			discontinued	south
Churchill River between Up and Low Muskrat		16	yes			discontinued	south
Eagle River above falls	10900	46	no	yes	4 months	active	south
Etamamiou River	2950	20	no	yes	none	discontinued	south
Joir River near boundary	2060	17	no			active	south
Kanairiktok River below Snegamook Lake	8930	18	no	yes	none	discontinued	north
Little Mecatina	19100	16				discontinued	south

Gauge Name	Drainage Area km^2	Years	Regulated	Continuous	Missing Data	State	North or South
Little Mecatina below Breton Lake	12100	16				discontinued	south
Little Mecatina River above Lac Fourmont	4540	34	no	no	7 months	active	south
Magpie River	7230	24	no	yes	none	active	south
Minipi River below Minipi Lake	2330	32	no	no	29 months	active	south
Moisie River	19000	37	no	no	18 months	active	south
Nabisipi River	2060	27	no	yes	none	discontinued	south
Naskaupi River at Fremont Lake	8990	16				discontinued	north
Naskaupi River below Naskaupi Lake	4480	32	no	no	28 months	active	north
Natashquan River	15600	22	no	yes	none	active	south
Natashquan River below East Natash River	11600	18	no	yes	none	discontinued	south
Romaine River	13000	46	no	yes	none	active	south
Saint Augustin River	5750	16				active	south
Saint Marguerite River	6140	51	yes			active	south
Saint Paul River	6630	35	no	no	12 months	active	south
Tonnerre River	684	40	no		>24 months	discontinued	south
Ugjoktok River below Harp Lake	7570	32	no	yes	none	active	north
Unknown river at Lake 51	19900	16	yes			discontinued	south

Of the 12 rivers remaining with suitable records, only three basins in northern Labrador met the criteria: Ugjoktok, Naskaupi and Kanairiktok. Of these three rivers, Ugjoktok was selected since it has the longer record of the three: 32 years versus 18 and 16 years for the other two rivers. The Ugjoktok River drains to the Labrador Sea and has a basin size of 7570km^2 above the hydrometric station, which is a moderate size for a gauged basin in Labrador.

There were more records available to choose from south of Goose Bay. Of the rivers draining towards the Gulf of St. Lawrence, Magpie, Romaine, Nabasipi, Aganus, Natashquan and Etamamiou remained after the initial screening. Most of these rivers had 20-27 years of data, however, the Romaine River had

46 years of data and up to 2010 was unregulated. As a result, Romaine was selected for further study. The basin size of the Romaine River above the hydrometric station is 13,000 km^2.

Of the remaining rivers draining toward the Labrador Sea, the Eagle and Alexis are located south of Goose Bay. They both have more than 30 years of data, are not regulated and the data are mostly continuous. The hydrometric station on the Alexis River has a basin size of 2310 km^2 while the Eagle River station has a basin size of 10,900 km^2, which is similar in size to the Romaine River station basin. Since one of the criteria was to have a range of drainage sizes, the Alexis River was selected in preference to the Eagle River because its drainage area above the station is small compared with that of Ugjoktok and Romaine. The Alexis, Romaine and Ugjoktok Rivers were therefore selected for detailed study.

2.3 Final Selection of Streamflow Series

In general, monthly streamflow series from one small, one medium and one large river station basin were selected. Each river has more than 32 years of predominantly continuous data with no regulation. One river is located north and two are located south of Goose Bay, with the two southern rivers having differing basin sizes, aspects and drainage outlet locations. This diversity of location, basin orientation and outlet location was selected to cover a wide variety of features that might provide a more varied list of rivers to model. Table 2-2 summarizes the information for these selected rivers and their locations are shown in Figure 2-1.

Table 2-2: Selected Rivers for Time Series Modelling

River	Drainage Area upstream of station (km^2)	Years of Data	Location	Drainage Basin Aspect	Outlet location
Ugjoktok	7,570 (medium)	32	North	West/East	Labrador Sea
Alexis	2,310 (small)	34	South	West/East	Labrador Sea
Romaine	13,000 (large)	56	South	North/South	Gulf of St. Lawrence

Chapter 3 Methodologies for Time Series Analysis

Two methodologies for modelling and predicting stream flows were used in this study: Box Jenkins Auto Regressive Moving Average (ARMA) and State-Space Time-Series (SSTS). The software Structural Time Series Modelling Program (STAMP Version 8.3) developed by Koopman et al (2009) and sold by Timberlake Consultants Ltd. was used to complete the state-space time-series modelling, as this is an accepted software package for SSTS modelling. ARMA modelling was completed using the Minitab 16 software (Minitab Inc, 2013), an accepted statistical software package for Box Jenkins applications. The goal was to compare the two modelling methodologies to determine accuracy of the model fit as well as prediction accuracy.

When modelling monthly flow time series, the fundamental characteristics that should be reproduced in the model are trend, seasonality and stochasticity. Testing for the presence of these characteristics was key to developing a model that could reproduce the actual data set as well as predict with accuracy. The Box Jenkins and SSTS methods handle trend, seasonality and stochasticity differently as is discussed in more detail in the following sections.

3.1 Box Jenkins Time Series Analysis

Box Jenkins methodology has been used extensively in hydrology for analysis of time series data with multiple examples available including Berlando et al (1993), Valipour et al (2013), Liu et al (2015) and Mondal and Wasimi (2007). These papers indicate the prevalent use of the Box Jenkins method in the study of rainfall, streamflow and climate.

Three specific modelling methods for conducting Box Jenkins Time Series analysis were examined prior to model development: Seasonal Autoregressive Moving Average with differencing (SARIMA), deseasonalized ARMA and periodic autoregressive (PAR). PAR is developed by fitting a separate autoregressive model to each month while ARIMA addresses seasonality through a differencing factor.

The deseasonalized ARMA includes removing the seasonality from the data set and applying ARMA to the nonseasonal data.

ARIMA modelling cannot be used for simulation (Hipel and McLeod, 1994), and since simulation is of interest for estimation of energy production and reliability, this Box Jenkins method was not considered for this study. PAR modelling of monthly time series can be used for simulation and forecasting, however it requires the use of a substantial number of parameters (Salas et al, 1980). In general for monthly time series, PAR results in the production of up to twelve autoregressive models, one for each month, which produces an overall model that is complex. Deseasonalized ARMA involves one model to deseasonalize the data and one ARMA model, reducing the complexity of the model compared with PAR.

A deseasonalized ARMA approach is preferred to a seasonal ARMA, like SARIMA, since when modelling monthly stream flows with seasonality, Box Jenkins models provide better results when the data set is first deseasonalized (Hipel and McLeod 1994). This may be in part due to the fact that a data set needs to be stationary when using the ARMA method, and seasonal data is not stationary. Deseasonalizing hydrologic data creates a residual data set that is stationary. Two advantages of using deseasonalized ARMA is that it can be easily used for forecasting and simulation (Hipel and McLeod, 1994) and produces a model that is not as complex as a PAR model, therefore, this method was selected for the Box Jenkins model development.

When using the deseasonalized ARMA approach, there are two ways to deseasonalize a time series: remove the seasonal mean and possibly the seasonal standard deviation from each data point in the time series or use Fourier Series (spectral analysis) to deseasonalize (Hipel and McLeod 1994). Subtracting the mean and standard deviation from each observation point results in a large number of model parameters. Fourier series is a way to remove seasonality while using fewer model parameters (Hipel & McLeod 1994). For this reason, the seasonal component for each time series was modelled using spectral analysis and the residuals (deseasonalized data) were then modelled using the ARMA methodology.

The premise of spectral analysis is that a periodic time series can be decomposed into a combination of sinusoidal patterns. To remove the seasonality using Fourier Series, the periods were first confirmed using the spectral density function (SDF). Spectral density function is a way to determine the more prominently recurring frequencies of pattern in the data set. Given a stationary time series, the spectral density function equation is as shown in Eqn [1]

$$I(f_j) = \frac{2}{N}\left[\left(\sum_{t=1}^{N} z_t \cos 2\pi f_j\, t\right)^2 + \left(\sum_{t=1}^{N} z_t \sin 2\pi f_j\, t\right)^2\right]^{\frac{1}{2}}$$

[1]

where $f_j = \frac{j}{N}$ is the jth frequency, j=1,2,.....N', N' = [N/2], z_t is the time series and t is the month. $I(f_j)$ measures the strength of the relationship between the time series and a sinusoid with frequency f_j

The spectral analysis (Jenkins and Watts, 1968) confirmed the periods and thus also the maximum number of sine and cosine pairs that could be applied in the Fourier analysis. For example, if the longest period is determined to be 12 months from the spectral analysis, the maximum number of sine and cosine pairs that can be used in the Fourier analysis is 6 pairs.

Once the periods for each river were confirmed, the seasonality was modelled using Fourier analysis by fitting a regression equation using sine and cosine pairs (Box et al, 2008). Eqn [2] represents the general Fourier regression equation.

$$Zt \;=\; \alpha \;+ \sum_{k=1}^{m} [\mathrm{A}_k \sin(\mathrm{k} * 2\pi f t) + \mathrm{B}_k \cos(\mathrm{k} * 2\pi f t)] + y_t$$

[2]

where Z_t is the time series, α is the regression constant, A_k is the coefficient for each sine term, B_k is the coefficient for each cosine term, $2\pi f$ is a constant with f = 1/12 for monthly values, t is the month, k = 1,2,…m for each sine and cosine pair and y_t are the residuals.

Three error formulas were used to determine the fit of the Fourier model; Nash Sutcliffe Efficiency Coefficient (NSE) (Moriasi et al, 2007), Mean Squared Deviation (MSD) and Median Absolute Percentage Error (MAPE). Eqns [3], [4] and [5] represent NSE, MSD and MAPE respectively.

$$NSE = 1 - \left[\frac{\sum(Q_{obs} - Q_{sim})^2}{\sum(Q_{obs} - Q_{\mu})^2}\right]$$

[3]

$$MSD = \frac{\sum(Q_{obs} - Q_{sim})^2}{n}$$

[4]

$$MAPE\% = Median\ of\ ABS\left|\frac{(Q_{obs} - Q_{sim})}{Q_{obs}}\right|$$

[5]

Where Q_{obs} is the observed flow, Q_{sim} is the simulated or fitted flow, Q_{μ} is the mean of the observed flows and n is the number of observations. The closer the NSE values are to 1 and the lower the MAPE and MSD numbers, the better the fit of the Fourier model values to the observed data set.

The residuals from the selected Fourier model are the deseasonalized flow data. This data was then used to develop the ARMA model. An ARMA model consists of an autoregressive (AR) process and a moving average (MA) process. The general equation for *p* AR parameters and *q* MA parameters is in Eqn [6]

$$(y_t - \mu) - \Phi_1(y_{t-1} - \mu) - \Phi_2(y_{t-2} - \mu) - \cdots - \Phi_p(y_{t-p} - \mu) = \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2} - \cdots - \theta_q\varepsilon_{t-q}$$

[6]

where y_t is the residual from Eqn [2], a time dependent series, μ is the mean of y_t, Φ is the AR coefficient, θ is the MA coefficient and ε_t is the residual of the ARMA process. A stipulation of the applicability of this equation, is that the residuals must be independent, stationary, homoscedastic and normally distributed. To check for independence, the Autocorrelation and Partial Autocorrelation Functions were plotted to determine if the remaining autocorrelations were within the 95% confidence limits and thus independent. A normality plot was used to test for normally distributed residuals and plots of residuals versus order and fits were reviewed to confirm homoscedasticity.

Flows were simulated to verify the ARMA model, demonstrating each model could indeed generate data sets of the same sample size that on average have similar historical statistics. The mean, standard deviation and lag 1 correlation (r1) for the generated data should be within the 95% confidence limits of the actual statistics. A Monte Carlo simulation was used to generate multiple sample sets and determine the average statistics of the generated data sets.

The Fourier model was used to forecast up to 12 months. For this study, the last five years of actual streamflow data were reserved from each modelled data set and compared to forecasts to determine forecasting accuracy of the models. The forecasted values were compared to the actual values using NSE, MSD and MAPE as was used to verify the Fourier model accuracy.

3.2 State-Space Time Series Analysis

The State Space Time Series method has been used in economic time series analysis as noted in Yang et al (2014), Nielsen and Berg (2014) and Balke et al (2013). Extensive research has revealed only a few papers relating state–space time series analysis to hydrologic time series. These include Mendelssohn (2011), where a rainfall time series for the Nile River was used as an example, and Sidhu (1995), where state-space analysis was compared to other time series modelling methods.

The State Space Time Series analysis is one whereby a series of state equations are used to explain individual components that together describe a time series (Harvey, 1989). The components include level (roughly equivalent to intercept in a regression equation), slope (together with level these form a trend), seasonality, cycle and irregular or white noise. Each component can be specified as deterministic or stochastic as required, meaning the component baseline is either constant or changing over time. A general univariate state-space time series model, using matrix algebra, can be stated as follows, whereby Eqn [7] is known as the observation equation and Eqn [8] is known as the state or transition equation.

$$Z_t = Z'_t \alpha_t + \varepsilon_t \qquad [7]$$

$$\alpha_{t+1} = T_t \alpha_t + R_t \eta_t \qquad [8]$$

where Z'_t is an (m x 1) observation vector, T_t is an (m x m) transition matrix, α_t is an (m x 1) state vector, m is the number of elements in the vector, R_t is an (m x m) identity matrix, η_t is the vector containing state disturbances and ε_t is a scalar disturbance term.

The above noted general equations can be written in scalar notation specific to the components selected for inclusion in the model. For example, for a SSTS monthly time series model with level, seasonal and irregular components, the observation equation is given by Eqn [9] and the state equations, for the same model, are given by Eqns [10] to [21] below.

$Z_t = \mu_t + \gamma_t + \varepsilon_t$ [9]

$\mu_{t+1} = \mu_t + \xi_t$ [10]

$\gamma_{1,\,t+1} = -\gamma_{1,t} - \gamma_{2,t} - \gamma_{3,t} - \gamma_{4,t} - \gamma_{5,t} - \gamma_{6,t-} - \gamma_{7,t} - \gamma_{8,t} - \gamma_{9,t} - \gamma_{10,t} - \gamma_{11,t} - \omega_t$ [11]

$\gamma_{2,\,t+1} = \gamma_{1,t}$ [12]

$\gamma_{3,\,t+1} = \gamma_{2,t}$ [13]

$\gamma_{4,\,t+1} = \gamma_{3,t}$ [14]

$\gamma_{5,\,t+1} = \gamma_{4,t}$ [15]

$\gamma_{6,\,t+1} = \gamma_{5,t}$ [16]

$\gamma_{7,\,t+1} = \gamma_{6,t}$ [17]

$\gamma_{8,\,t+1} = \gamma_{7,t}$ [18]

$\gamma_{9,\,t+1} = \gamma_{8,t}$ [19]

$\gamma_{10,\,t+1} = \gamma_{9,t}$ [20]

$\gamma_{11,\,t+1} = \gamma_{10,t}$ [21]

where μ_t is the level component at time *t*, γ_t is the seasonal component with (*s*-1) state equations where *s* is the periodicity of the seasonal component, ξ_t is the level disturbance, ε_t is the irregular or white noise component of the time series, ω_t is the seasonal disturbance, and Z_t is the time series. The observations and disturbances are assumed to be independent and normally distributed.

The STAMP software applies the state-space time-series modelling theory described in Harvey (1989) including filtering and prediction using the Kalman filter. The Kalman filter is a procedure that is used for computing the optimal values of the state at time, t (Harvey, 1989). It enables the estimate of the state vector to be continually updated as new observations become available to optimize the model.

In order to select the best fitting SSTS model, the model results must be reviewed and the diagnostic test values must fit within critical values. Assumptions for independence, homoscedasticity and normality of the model residuals must be met. Diagnostic tests are reviewed to determine if these assumptions are met and include the following:

1. Independence:
    - Autocorrelation at lag 1, r(1), and autocorrelation at lag q, r(q), must stay within the 95% confidence limits of $\pm 2/\sqrt{n}$, where n is the sample size,
    - Q-test number, Q(k) should be less than the chi statistic $\chi^2_{(k-w+1;\,0.05)}$, where k is the specified lag and w is the number of estimated disturbance variances,

2. Homoscedasticity:

   - H-statistic, H(h), should be equal to $F_{(h,h;0.025)}$, where h is the degrees of freedom. This is a two tailed test thus for a p=0.05, the alpha value in the F distribution is 0.025,

3. Normality:

   - N-statistic tests whether the skewness and kurtosis of the residual distribution meet a normal Gaussian distribution. N-statistic should be less than $\chi^2_{(2;0.05)}$

The Akaike Information Criterion (AIC) is used to compare the various state space models. The AIC equation is shown in Eqn[22].

$$AIC = \frac{1}{n}[-2n\log L_d + 2(q + w)]$$

[22]

where n is the number of observations, $\log L_d$ is the log-likelihood function (Commandeur and Koopman, 2007), q is the number of initial values in the state and w is the number of initial variances.

The most appropriate model is used to forecast the time series, by continuing the Kalman filter prediction beyond the end of the observed data set.

Chapter 4 Application of Time Series Methodologies to Selected Rivers

In this chapter, preliminary analysis of streamflow records from the Alexis, Ugjoktok and Romaine Rivers provided details of the characteristics of each time series. Once characteristics were identified, the streamflow data were used to develop both a Box Jenkins model, in this case a deseasonalized ARMA, as well as a SSTS model for each river. Flows were forecasted using these models and reviewed for prediction accuracy against the reserved set of actual flows for each river.

4.1 Preliminary Analysis

Preliminary analysis of the data helps show important features of the data set. Such concepts as stationarity, homoscedasticity, periodicity, normality and autocorrelation can be identified through the use of data plots and diagnostic tests. Following are the preliminary analysis results for each of the three rivers in the study.

4.1.1 Alexis River

The Alexis River is located in southern Labrador. As shown in Table 2-2, the basin size upstream of the EC gauge is 2310 km^2. There is an Environment Canada stream gauge, located within 500m of a potential hydro site, which has been in operation since 1978. Monthly streamflow data were collected from this EC gauge for the period of 1978 to 2010 and the data set was analyzed using time series analysis. As with all streams and rivers in Labrador, seasonality was expected.

The summary statistics for the data set are shown in Table 4-1 below where Q1 is the flow at the lower quartile, Q3 is the flow at the upper quartile and r1 is the lag 1 correlation value.

Table 4-1: Summary Statistics for Alexis River Monthly Streamflow Data

N	Mean	Std dev	Min Flow	Max Flow	Skew	Median	Q1	Q3	r1
396	51.72	62.63	3.07	327.00	2.13	30.60	11.03	57.72	0.2852

Box plots of the data set, Figure 4-1, as well as the data set by month, Figure 4-2, indicated the flows were not normally distributed and as such the data set was transformed using a natural log transformation. The box plots of the Log-transformed data set, as well as the data set per month, are shown in Figure 4-3 and Figure 4-4 below. Flows are in m^3/s.

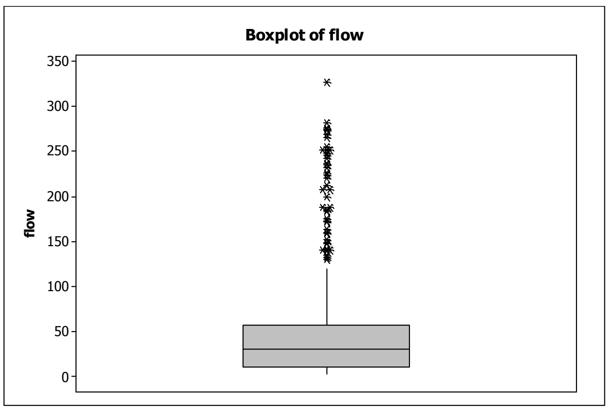


Figure 4-1: Box Plot of Alexis River Flows

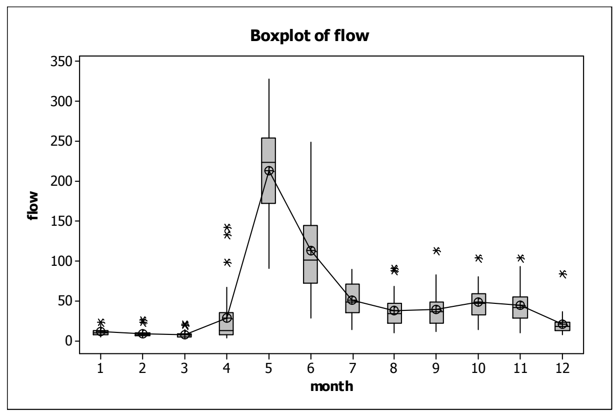


Figure 4-2: Box Plot by Month of Alexis River Flows

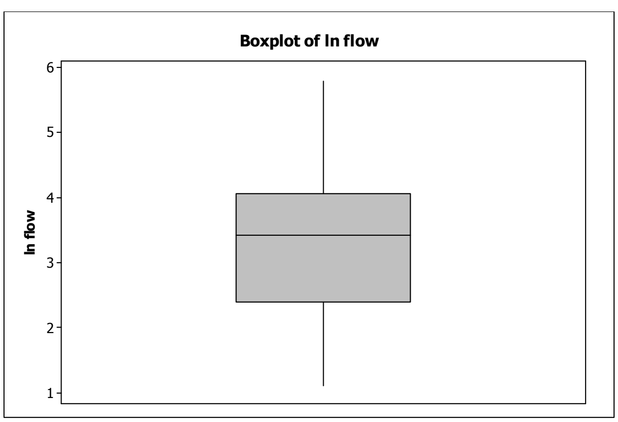


Figure 4-3: Box Plot of Log-transformed Data Set for Alexis River

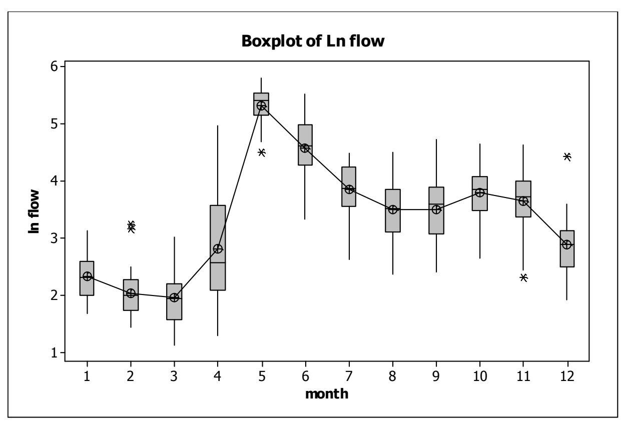


Figure 4-4: Box Plot by Month of Log-transformed Data for Alexis River

As shown in Figure 4-3, the Log-transformed data set appears to be approximately normally distributed as evidenced by the relatively equidistant whiskers on both sides of the quartile box. In the box plot by month of the Log-transformed data, Figure 4-4, potential outliers still appear in the monthly data, however, this box plot is useful as it illustrates there is periodicty in the data as evidenced by the changing monthly mean. As the data being analyzed is hydrological data, we can presume this periodicity is seasonal with an increase in the mean around May during spring flooding and a decrease around January to March when the river is frozen. The presence of periodicity indicates the data set is not stationary.

Normality plots were completed for the original data and the Log-transformed data. As shown in Figure 4-5 and Figure 4-6 below, the Ryan Joiner (RJ) normality test value is closer to 1 in Figure 4-6, indicating the data better fit the lognormal distribution, so this transformation was applied in further analysis.

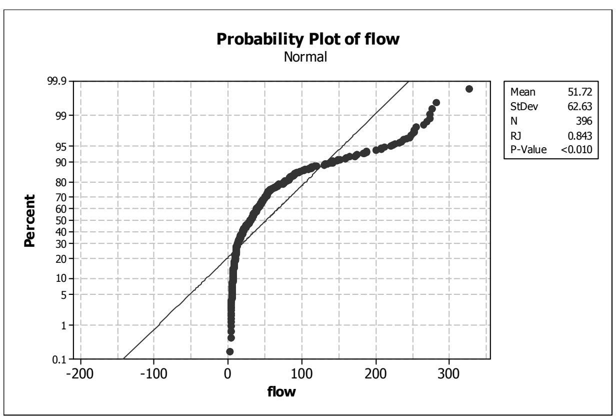


Figure 4-5: Normality Plot of Original Alexis River Data Set

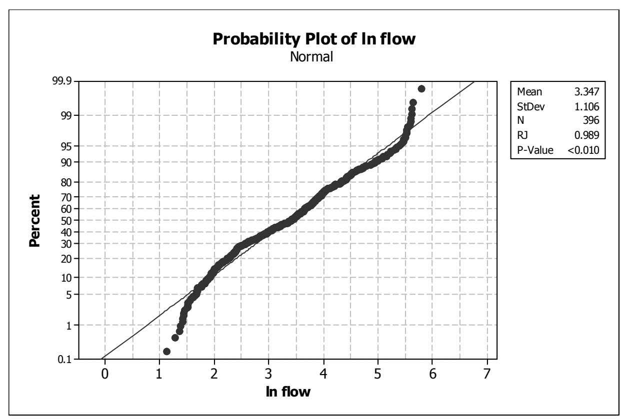


Figure 4-6: Normality Plot of Log-transformed Alexis River Data Set

The time series plot with LOWESS smoothing as well as a fitted line plot were used to determine if there was a trend in the data set. As shown in Figure 4-7 and Figure 4-8 below, the time series plot and the fitted line plot suggest there is no trend in the data.

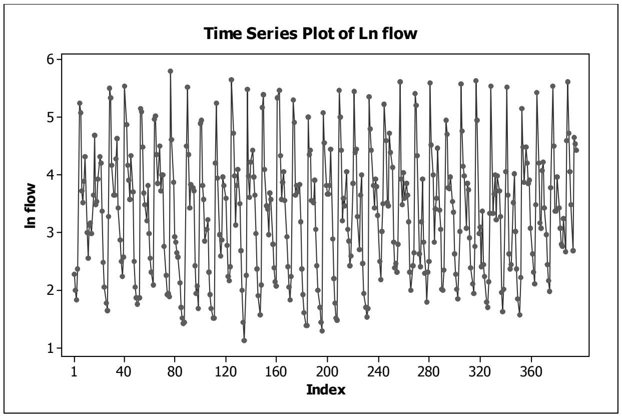


Figure 4-7: Time Series Plot of Log-transformed Alexis River Data Set

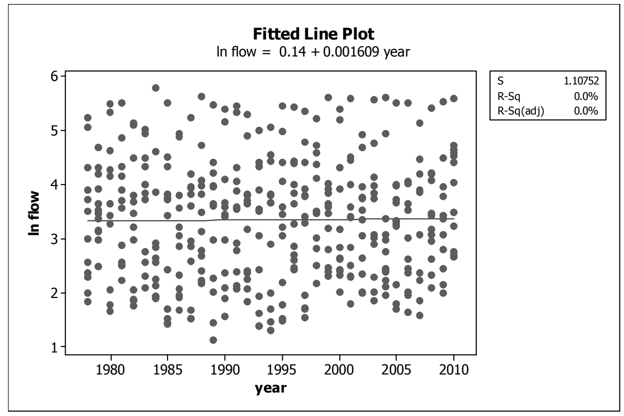


Figure 4-8: Fitted Line Plot of Log-transformed Alexis River Data Set

The results of the fitted line plot regression analysis using Minitab are shown in Table 4-2. The null hypothesis for this regression analysis is that trend is significant using a test statistic of 5% or 0.05. A review of the regression results below indicates that the P value of the regression is 0.78, which is greater than the test statistic value of 0.05 and thus the null hypothesis is rejected. The conclusion is that trend is not statistically significant.

Table 4-2: ANOVA Table of Fitted Plot Trend Line for Alexis River Data

Source	Sum of Squares	df	Mean Square	F Value	P Value Prob > F	
Regression	0.093	1	0.09291	0.08	0.78	Not significant
Error	483.279	394	1.22660			
Total	483.372	395				

The assumptions of ANOVA, in which the residuals are normally distributed, independent and with constant variance, were met, as shown in Figure 4-9 below.

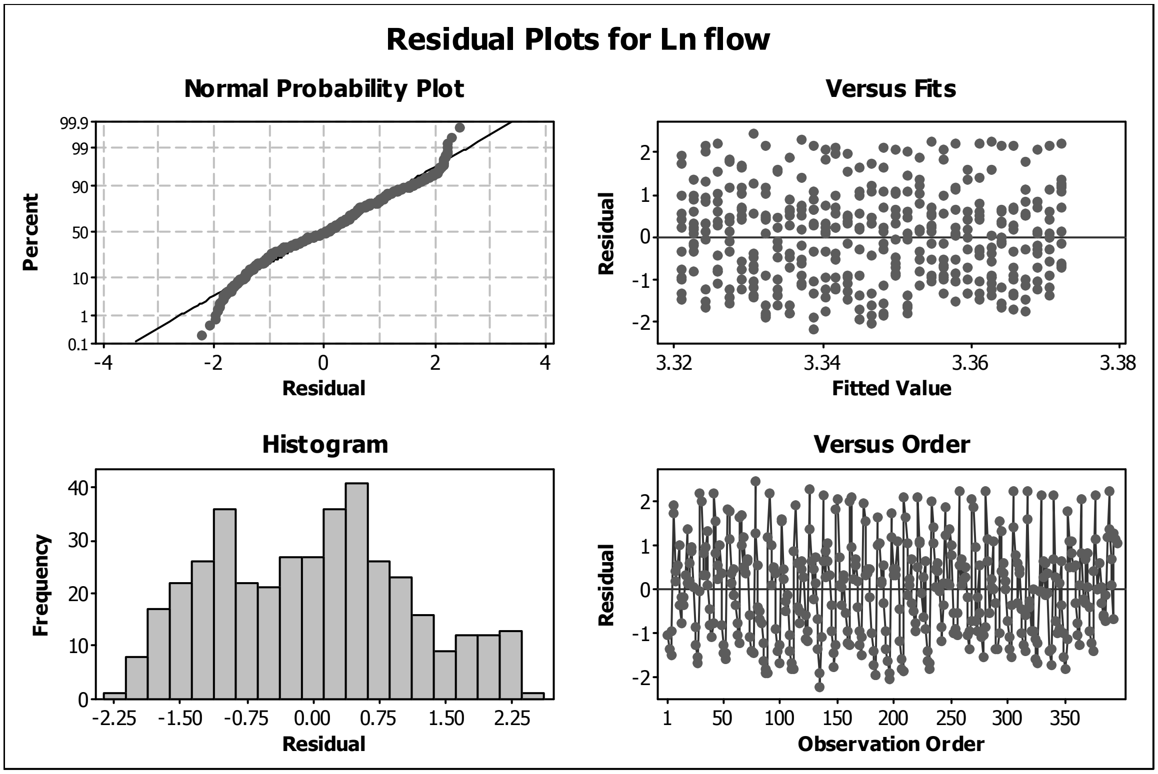


Figure 4-9: Residual Test Plots for Fitted Line Regression for Alexis River Flow

The value of an observation can be dependent on the value of previous and adjacent observations. This relationship is called short term dependence and was reviewed for this data set. An autocorrelation function (ACF) plot was completed to test for correlation between values at different lag times in the time series, and as shown in Figure 4-10, indicates that correlation between the flows was significant. This is evidenced by multiple autocorrelation values greater than the dashed significance line in the ACF plot. Periodicity was also confirmed by the presence of a sine wave appearance in the plot. A review of the partial autocorrelation function (PACF) plot, as shown in Figure 4-11, confirms there was significant correlation in the data set, again since autocorrelation values exceed the dashed significance line, and therefore there is short term dependence in the flows for the Alexis River.

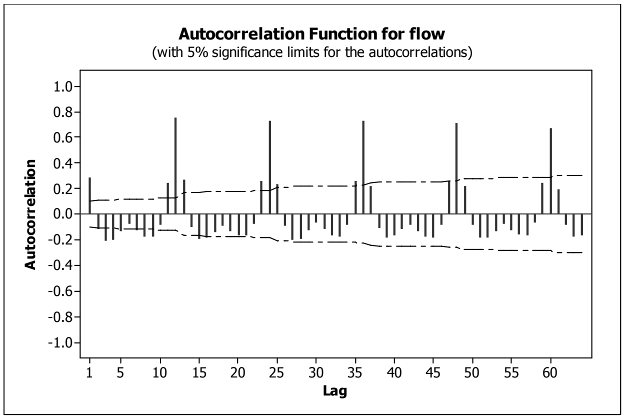


Figure 4-10: ACF Plot for Alexis River Flows

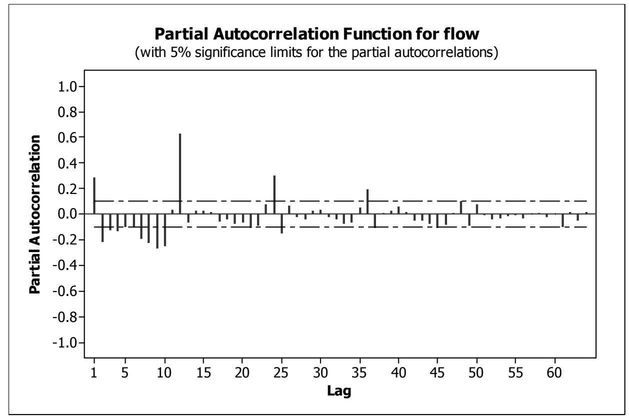


Figure 4-11: PACF Plot for Alexis River Flows

4.1.2 Ugjoktok River

Ugjoktok River is located north of the Churchill River in Labrador and flows west to east toward the Labrador Sea. The basin size above the stream gauge is 7570 km^2 and the gauge is currently active with 32 years of data available, from 1979 to 2010. The river is not regulated but there is a three month gap in the monthly data set in 1981. Data from the Kanairiktok River, which is located in proximity to the Ugjoktok River, were used to populate the missing data at Ugjoktok using Line of Organic Correlation (LOC) since this method is able to maintain variability in extended records (Helsel and Hirsch, 2002). The standard deviation and the mean from the Kanairiktok River data as well as the standard deviation and mean of the Ugjoktok River flow data were used to infill the missing values in the Ugjoktok data set. Eqn [23] is the general equation that was used to infill the missing data for months 9, 10 and 11 in 1981 for Ugjoktok.

$$\mathbf{Y} = \left[\boldsymbol{\mu}\mathbf{a} - \left(\frac{\boldsymbol{\sigma}\mathbf{a}}{\boldsymbol{\sigma}\mathbf{b}}\right) * \boldsymbol{\mu}\mathbf{b}\right] + \mathbf{r} * \left(\frac{\boldsymbol{\sigma}\mathbf{a}}{\boldsymbol{\sigma}\mathbf{b}}\right) * \mathbf{X} \qquad [23]$$

Where μa is the mean of the data set with missing data (Ugjoktok),

μb is the mean of the reference data set (Kanairiktok),

σa is the standard deviation of the data set with missing data (Ugjoktok), σb is the standard deviation of the reference data set (Kanairiktok),

r is +1 for positive slope and -1 for negative slope,

Y is the predicted infill data point, and

X is the reference data point (Kanairiktok)

The following equation, Eqn [24] is the LOC equation used to infill missing Ugjoktok River data using Kanairiktok data.

$$\mathbf{Y} = -\mathbf{0.825502} + \mathbf{1.12248} * \mathbf{X} \qquad [24]$$

Where Y is the natural log of the predicted infill data point (Ugjoktok river), and

X is the natural log of the reference data point (Kanairiktok river)

Table 4-3 lists the resulting infill data point values for the Ugjoktok River using Eqn [24].

Table 4-3: Summary of Ugjoktok Infilled Data Point Values using LOC

Year - Month	**Infilled Data Point for Ugjoktok River**	
	Ln Flow	**Flow (m^3/s)**
1981-9	5.06	150.79
1981-10	5.53	251.42
1981-11	4.66	105.14

The summary statistics for the data set are shown in Table 4-4.

Table 4-4: Summary Statistics for Ugjoktok River Monthly Data

N	Mean	Std dev	Min Flow	Max Flow	Skew	Median	Q1	Q3	r1
384	160.20	197.60	4.20	1100.00	2.29	96.90	28.30	199.50	0.3607

Box plots for the entire data set as well as by month, as shown in Figure 4-12 and Figure 4-13, indicate that the data are not normally distributed and therefore the data set was transformed using a natural log transformation. The box plots for the Log-transformed data set and the Log-transformed data by month are shown in Figure 4-12 and Figure 4-13 below.

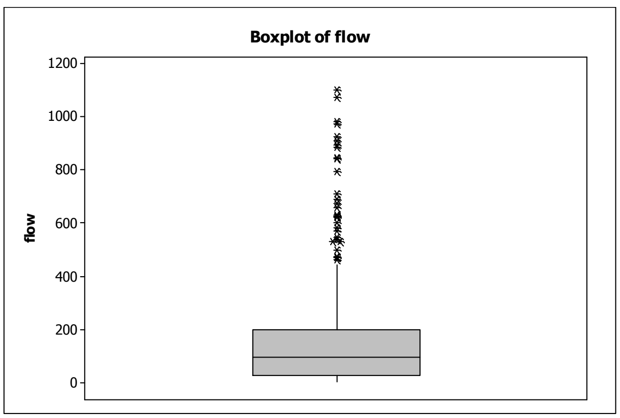


Figure 4-12: Box Plot of Data Set for Ugjoktok River

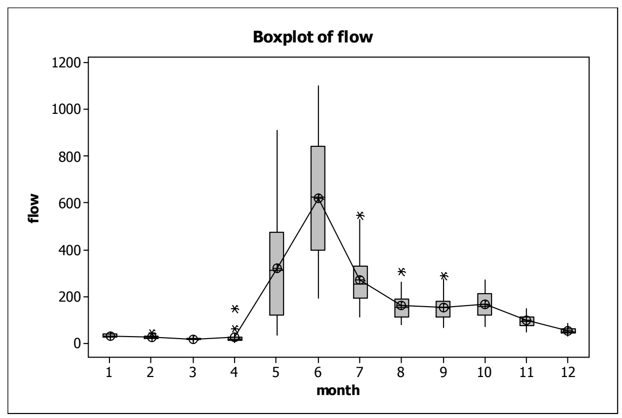


Figure 4-13: Box Plot by Month for Ugjoktok River

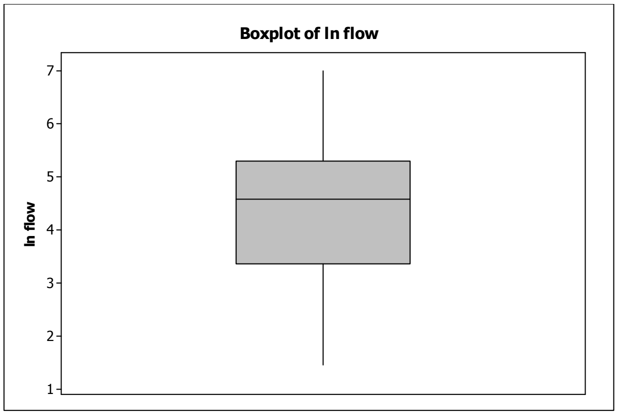


Figure 4-14: Box Plot of Log-transformed Data Set for Ujgoktok River

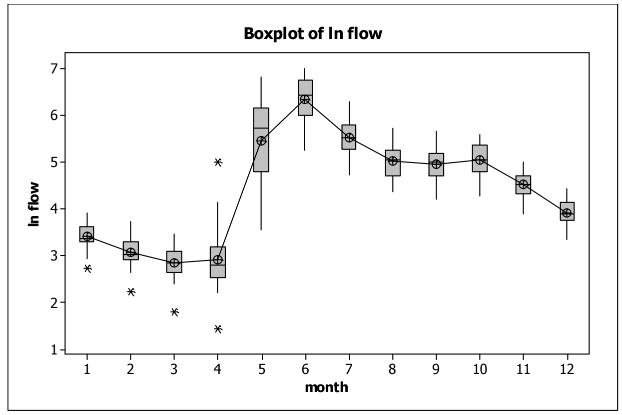


Figure 4-15: Box Plot by Month of Log-transformed Data for Ugjoktok River

As shown in Figure 4-14, the data set appears to be lognormally distributed. When the box plot of the Log-transformed data was plotted by month, Figure 4-15, potential outliers were still visible, however, the box plot whiskers and quartile boxes were generally more even and regular. This box plot confirms periodicity with an increase in the mean around May and a decrease around January to March, as was the case with the Alexis River flows.

Normality plots were completed for the original data and the Log-transformed data. As shown in Figure 4-16 and Figure 4-17 below, the RJ value for the lognormal data is closer to 1, indicating the data better fit the lognormal distribution. This transformation was applied in further analysis.

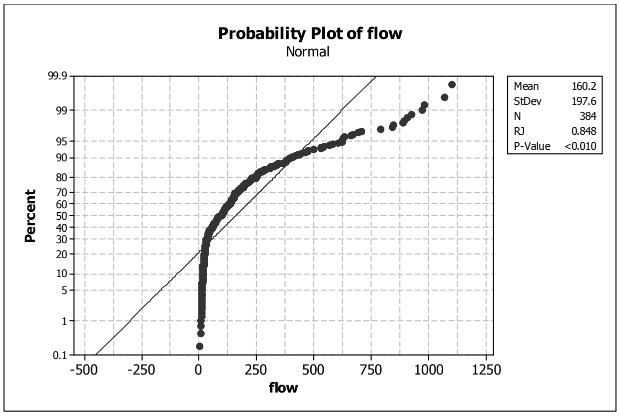


Figure 4-16:Normality Plot of the Original Ugjoktok River Data Set

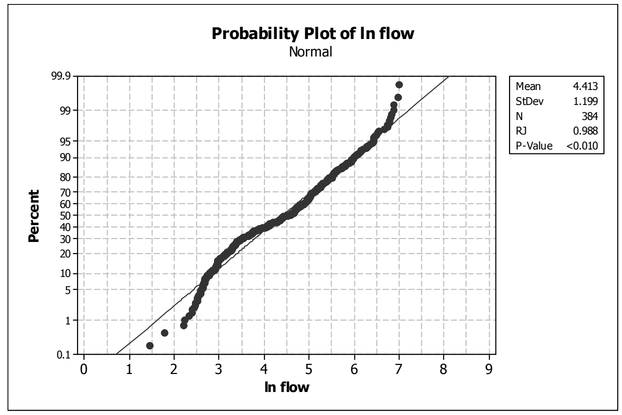


Figure 4-17: Normality Plot of the Log-transformed Ugjoktok River Data Set

The time series plot with LOWESS smoothing as well as a fitted line plot were used to determine if there was a trend in the Log-transformed data set. As shown in Figure 4-18 and Figure 4-19 below, the time series plot and the fitted line plot suggest there was no trend in the data.

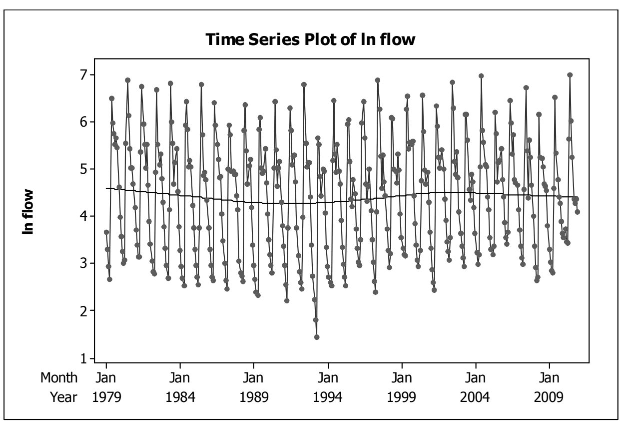


Figure 4-18: Time Series Plot of Log-transformed Ugjoktok River Data Set

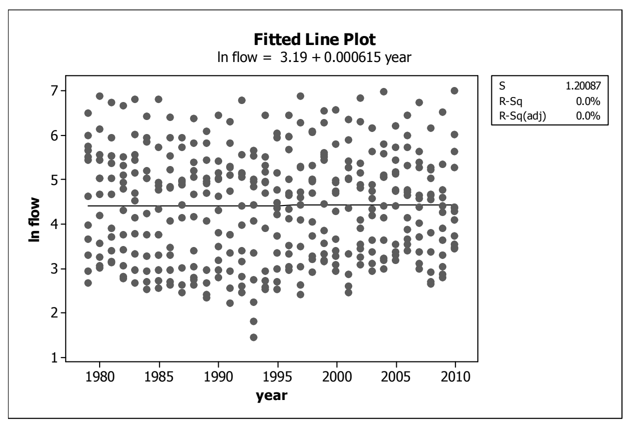


Figure 4-19: Fitted Line Plot of Log-transformed Ugjoktok River Data Set

A review of the regression analysis for the fitted line plot indicates there is no statistically significant trend in the data set. Shown below in Table 4-5, are the fitted plot regression results using Minitab.

Table 4-5: ANOVA Table for Ugjoktok River Fitted Plot Trend Line

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Regression	0.012	1	0.01236	0.01	0.93	Not significant
Error	550.878	382	1.44209			
Total	550.891	383				

The assumptions of ANOVA, in which the residuals are normally distributed, independent and with constant variance, were met, as shown in Figure 4-20 below.

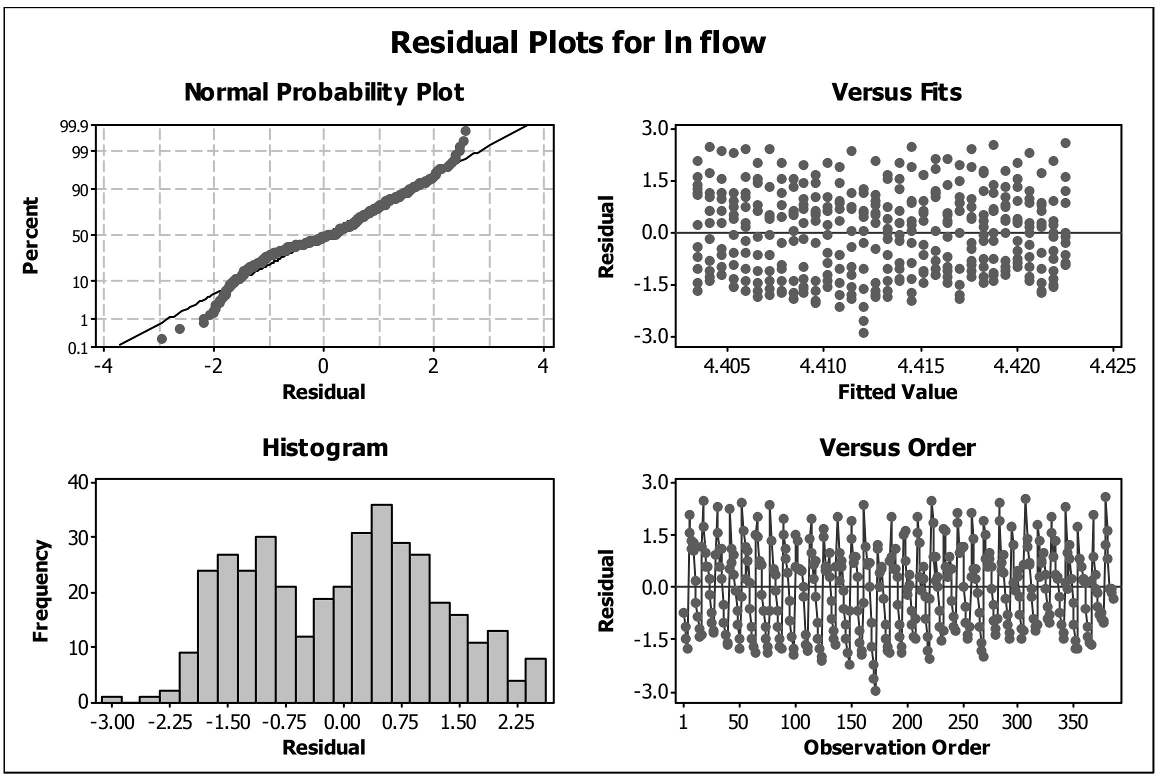


Figure 4-20: Residual Test Plots for Fitted Line Regression for Ugjoktok River Flow

An ACF plot was completed to test for correlation and short term dependence. Correlation was significant as shown in Figure 4-21 below. Periodicity was also confirmed by the presence of a sine wave appearance in the ACF plot. A review of the PACF plot, as shown in Figure 4-22 below, confirms significant correlation in the data set.

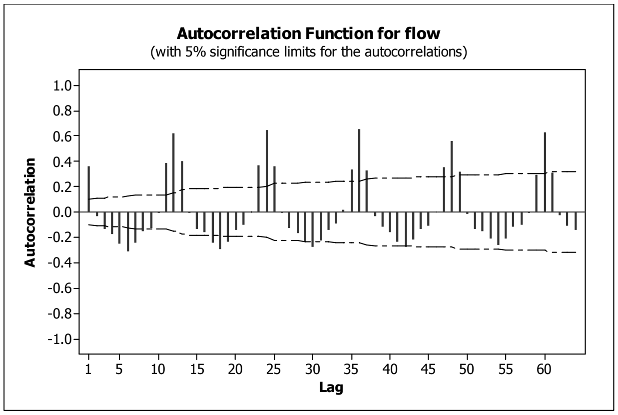


Figure 4-21: ACF Plot for Ugjoktok River Flow

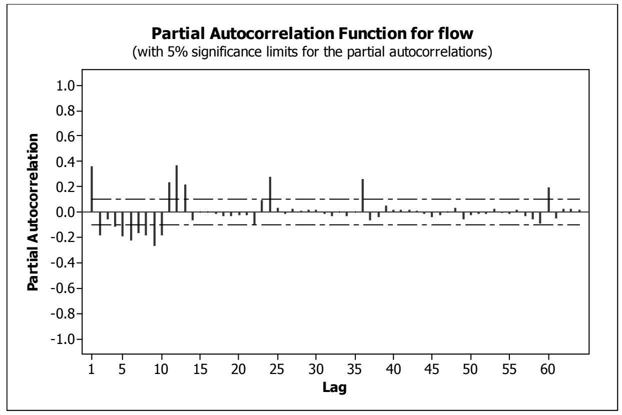


Figure 4-22:PACF Plot for Ugjoktok River Flow

4.1.3 Romaine River

Romaine River is located south of the Churchill River in Labrador and flows from north to south toward the Strait of Belle Isle. The basin size above the stream gauge is 12,922 km^2 and the gauge is currently active with 54 years of data used in the analysis, from 1956 to 2010. There is a gap in the 1959 data and as a result, the data set was truncated to exclude these values: the analyzed data starts in 1960. During the period of record, from 1960 to 2010, the river was not regulated. More recently, hydroelectric plants have been under construction on the Romaine River however, regulation of the river began after 2010.

There were 6 missing months in 2007. The nearby Natashquan River was used to populate the missing data at Romaine using LOC. The standard deviation and the mean from the Natashquan River flow data as well as the standard deviation and mean of the Romaine River flow data were used to infill the missing values in the Romaine data set. Eqn [23], as shown in section 4.1.2, was used to infill the missing data for months 4 to 9 in 2007 for Romaine.

The following equation, Eqn [25] is the LOC equation used to infill missing Romaine River data using Natashquan data.

$$\mathbf{Y} = -\mathbf{0.363673} + \mathbf{1.03079} * \mathbf{X} \qquad [25]$$

where Y is the natural log of the predicted infill data point (Romaine river), and

X is the natural log of the reference data point (Natashquan river)

Table 4-6 lists the resulting infill data point values for the Romaine River using Eqn [25].

Table 4-6: Summary of Romaine River Infilled Data Point Values using LOC

Year - Month	**Infilled Data Point for Romaine River**	
	Ln Flow	**Flow (m^3/s)**
2007- 4	4.68	107.48
2007- 5	6.64	766.09
2007- 6	6.43	621.66
2007- 7	6.07	431.28
2007- 8	4.95	141.78
2007- 9	5.24	189.02

Incorporating the infilled data points using LOC, the summary statistics for the data set are shown in Table 4-7 below.

Table 4-7: Summary Statistics for Romaine River Monthly Data

N	Mean	Std dev	Min Flow	Max Flow	Skew	Median	Q1	Q3	r1
604	290.40	264.40	36.30	1540.00	1.91	213.30	93.50	365.80	0.3996

Box plots for the entire data set as well as by month, as shown in Figure 4-23 and Figure 4-24, indicate that the data are not normally distributed and therefore the data set was transformed using a natural log transformation. The box plots for the Log-transformed data set and the Log-transformed by month are shown in Figure 4-23 and Figure 4-24.

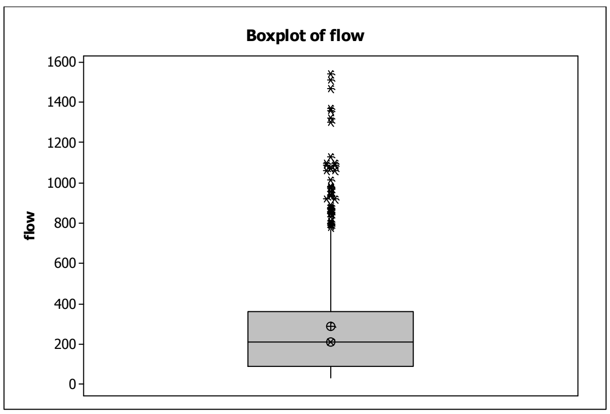


Figure 4-23: Box Plot of Data Set for Romaine River

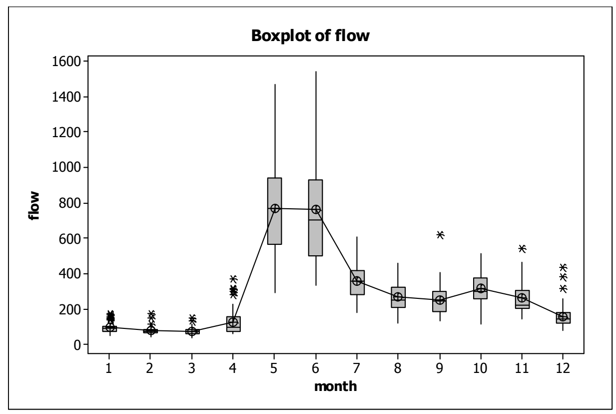


Figure 4-24: Box Plot by Month of the Romaine River Data Set

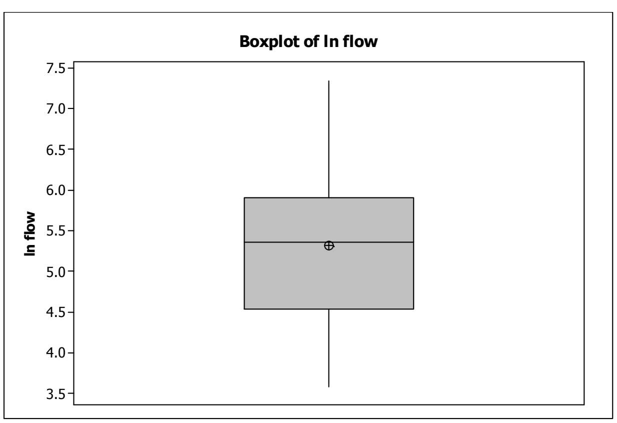


Figure 4-25: Box Plot of Log-transformed Data Set for Romaine River

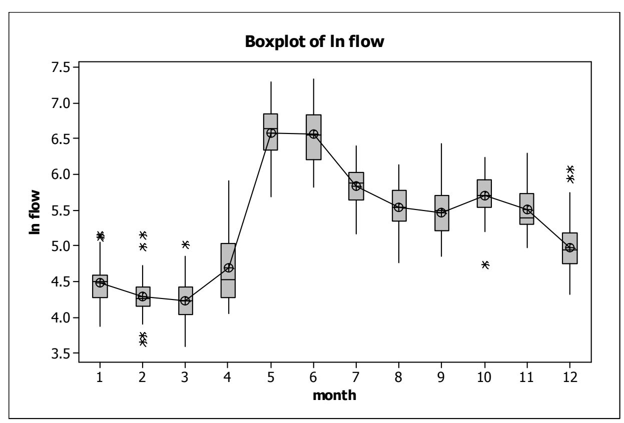


Figure 4-26: Box Plot by Month of Log-transformed Data for Romaine River

As shown in Figure 4-25, the data set appears to be approximately lognormally distributed. When the box plot of the Log-transformed data was plotted by month, Figure 4-26, potential outliers were still visible, however, the box plot whiskers and quartile boxes were generally more even and regular. This box plot confirms periodicity with an increase in the mean around May and a decrease around January to March, as was the case with the Alexis and Ugjoktok River flows.

Normality plots were completed for the original data and the natural Log-transformed data. As shown in Figure 4-27 and Figure 4-28 below, the data better fits the lognormal distribution, so this transformation was applied in further analysis.

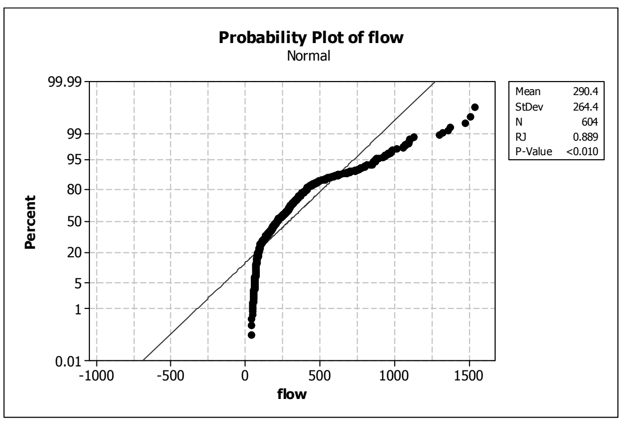


Figure 4-27: Normality Plot of Original Data for Romaine River

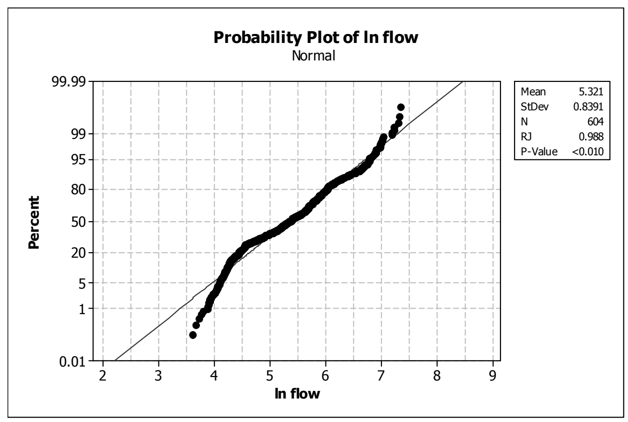


Figure 4-28: Normality Plot of Log-transformed Data for Romaine River

The time series plot with LOWESS smoothing as well as a fitted line plot were used to determine if there was a trend in the Log-transformed data set. As shown in Figure 4-29 and Figure 4-30 below, the time series plot and the fitted line plot suggest no significant trend in the data.

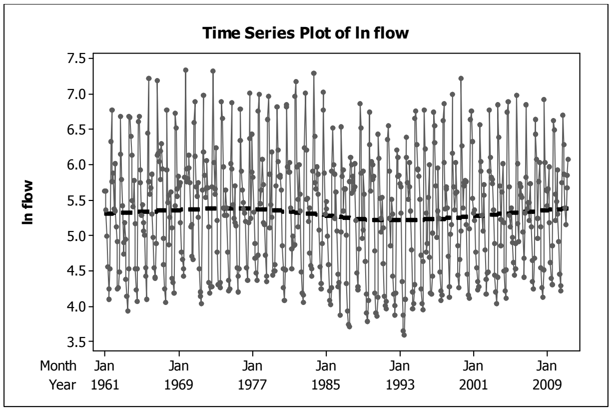


Figure 4-29: Time Series Plot of Log-transformed Data Set for Romaine River

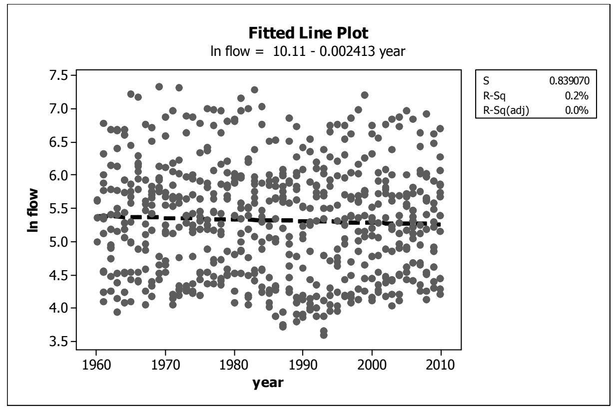


Figure 4-30: Fitted Line Plot of Log-transformed Data Set for Romaine River

A review of the regression analysis for the fitted line plot indicates there is no statistically significant trend in the data set. Shown below in Table 4-8, are the fitted plot regression results from Minitab.

Table 4-8: ANOVA Table for Romaine River Fitted Plot Trend Line

Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Regression	0.743	1	0.742518	1.05	0.31	Not significant
Error	423.831	602	0.704038			
Total	424.573	603				

The assumptions of ANOVA, in which the residuals are normally distributed, independent and with constant variance, were met, as shown in Figure 4-31 below.

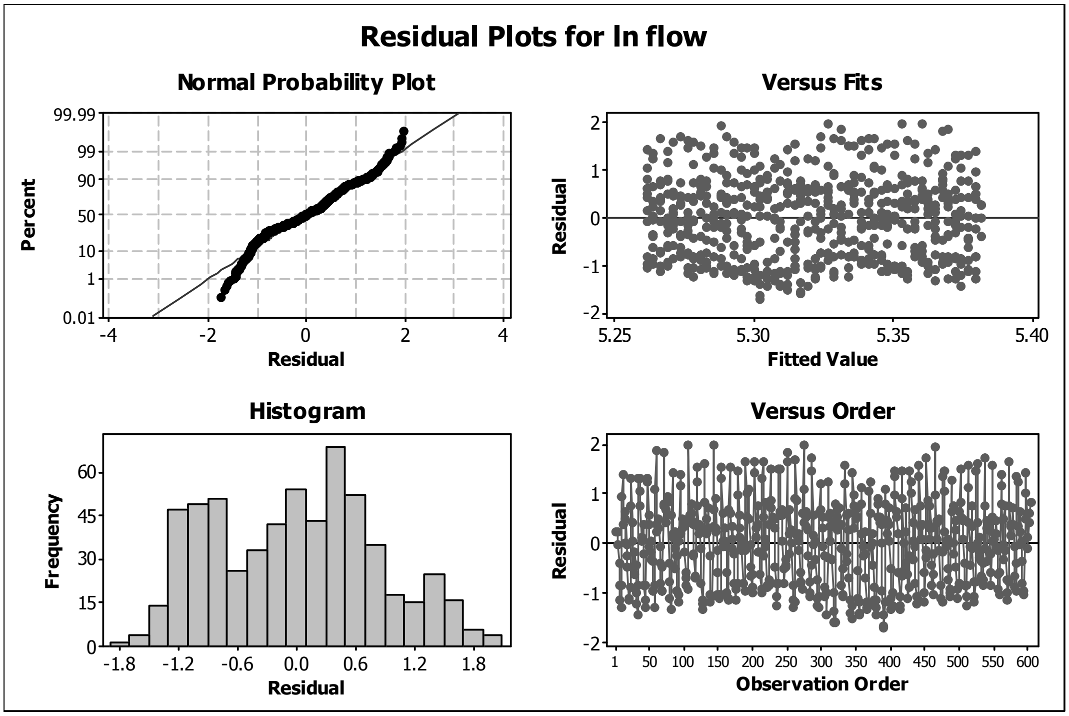


Figure 4-31: Residual Test Plots for Fitted Line Regression for Romaine River Ln Flow

Short term dependence of the data set was reviewed. An ACF plot was completed to test for correlation and as shown in Figure 4-32 below, indicates that correlation is significant. Periodicity was also confirmed by the presence of a sine wave appearance in the plot. A review of the PACF plot, as shown in Figure 4-33 below, confirms there is significant correlation in the data set and therefore there is short term dependence in the flows for the Romaine River.

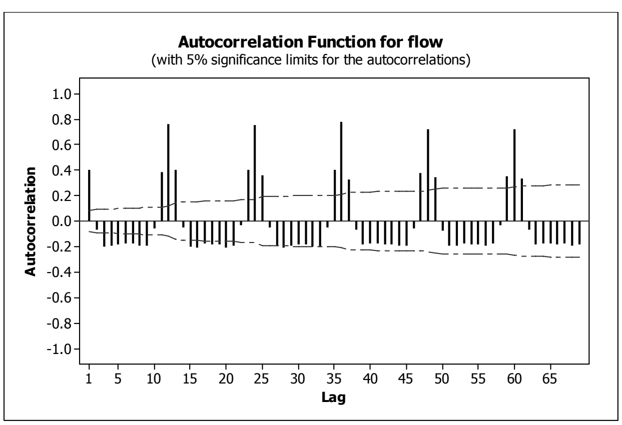


Figure 4-32: ACF Plot for Romaine River Flow

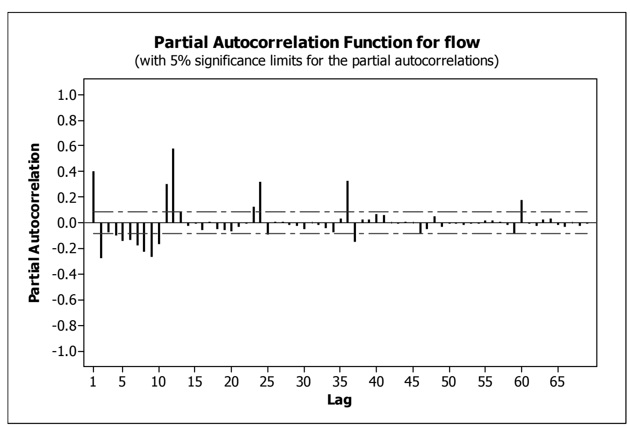


Figure 4-33: PACF Plot for Romaine River Flow

4.1.4 Summary

The preliminary analysis of the Alexis, Ugjoktok and Romaine Rivers indicated that although these rivers vary in station basin size, geographic location and drainage aspect, they were similar in some key characteristics. All three rivers displayed seasonality with highest flows occurring in May or June and lowest flows occurring around March month. The rivers also better fit a natural log transformation as evidenced by the Normality plots that were completed for both the original and Log-transformed data sets. In addition, none of the rivers displayed a statistically significant trend but did display short term dependence in the monthly flows.

4.2 Box Jenkins Time Series Analysis

As discussed in Chapter 3, the "Deseasonalized ARMA" modelling method was used in this study as the preferred Box Jenkins method for comparison with SSTS modelling. First steps in this methodology included completing a seasonal analysis of each river prior to modelling and separating the seasonal element from the flow data. Spectral analysis and Fourier Series analysis were used to identify and model the seasonal elements after which the deseasonalized data were modelled using ARMA to develop a suitable model for prediction.

In order to facilitate verification of the model predictions, the last five years of each actual time series data set was removed from the data sets used for model development. The removed actual data could then be compared to the forecasted data from each selected model, understanding that only short term forecasting is relevant, as forecasting more than 12 months ahead loses significance (Sidhu & Lye 1995).

4.2.1 Alexis River

Preliminary analysis of the Alexis River streamflows indicated that the data were not normally distributed as evidenced by a normality test and box plot, however, the data were found to be approximately normal after a log transformation. Box plots showed seasonality in the data and there was no significant trend detected in the fitted line test. Tests for autocorrelation revealed there is short term dependence in the flow data for the Alexis River. The data from 1978 to 2005 were used to develop the model, with the last five years of the actual data set, 2006 to 2010, being used to compare the model forecast values to actual values.

4.2.1.1 Seasonal Analysis

This time series model included modelling seasonality, removing that seasonality and modelling the remaining residuals. In order to model the seasonality, the periods associated with periodic data sets must be identified. Since the monthly Alexis River data were shown to have periodicity, the identification of the periods was accomplished using spectral analysis. A macro developed by Dr. Leonard Lye and modified by the author, as shown in Appendix A, was used to perform the analysis. A scatterplot of the spectral density function versus period is shown below in Figure 4-34. The plot suggests there are three periods in the data set; one at 12 months, one at 6 months and one at 4 months. Figure 4-35 is a scatterplot of the spectral density function versus frequency. This plot indicates there are three frequencies; one at 0.08, one at 0.17 and one a 0.25. Since the period is the inverse of frequency, the noted frequencies correspond to periods of 12 months, 6 months and 4 months as identified in Figure 4-34.

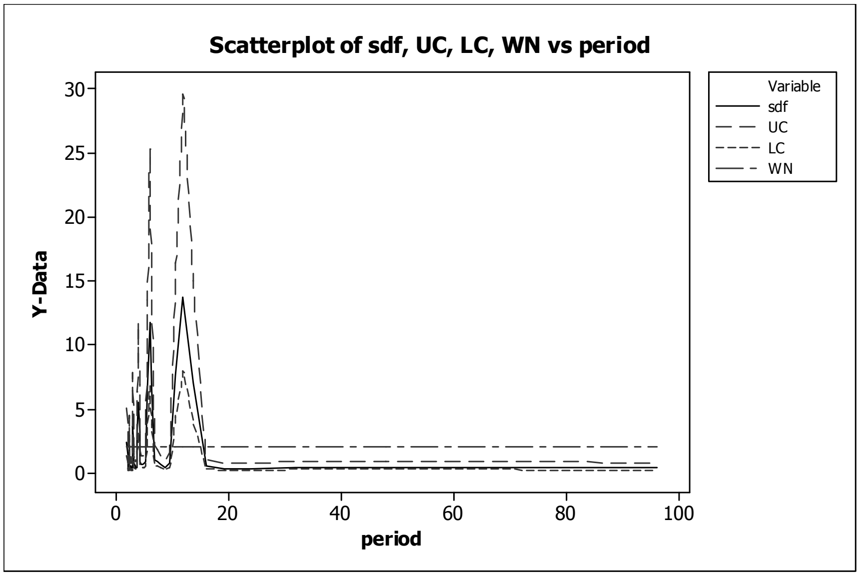


Figure 4-34: Scatterplot of Spectral Density Function versus Period for Alexis River

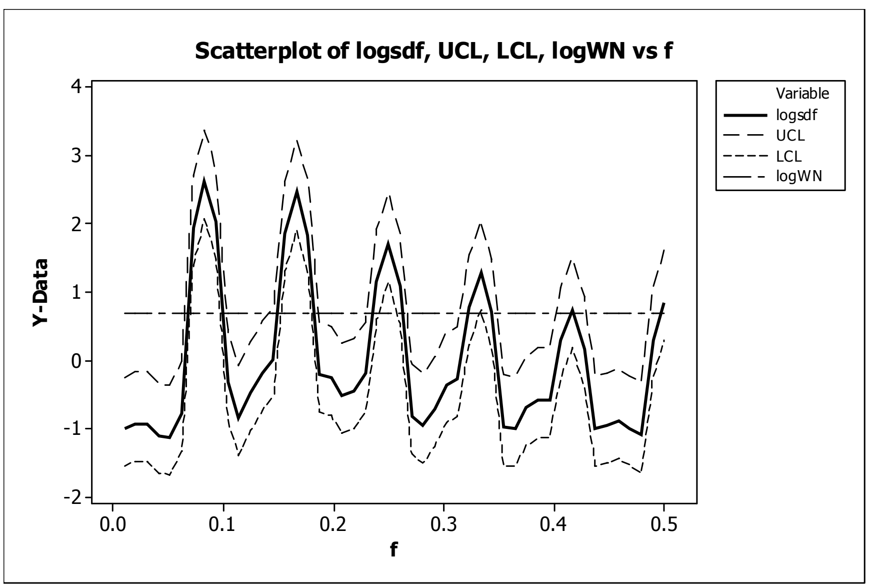


Figure 4-35: Scatterplot to Spectral Density Function versus Frequency for Alexis River

With seasonality and the periods confirmed by spectral analysis, the seasonal component of the data set was modeled using the Fourier Series method. The data was modeled using 1, 2, 3, 4, 5 and 6 sine and cosine pairs and the adjusted R^2 term was used to determine the best fitting equation. Analysis proved that the best fitting equation included 5 sine and cosine pairs, where all the pairs were significant, the residual error was low and adjusted R^2 value was the highest of all the models at 75.1%. As a result, the Fourier 5 pair model was selected. The results of the regression analysis are shown below in Table 4-9 and Table 4-10. The predictors listed in the table form part of the regression equation and in addition to a constant value, include sine and cosine pairs, indicating the ultimate number of pairs in the regression equation.

Table 4-9: Regression Results for Fourier 5 Sine and Cosine Pair Model for Alexis River

Predictor	**Coefficient**	**SE Coefficient**	**T**	**P**	
Constant	3.32951	0.02928	113.71	0.000	Significant
Sin(2pt)	-0.63006	0.04141	-15.22	0.000	Significant
Cos(2pt)	-0.81359	0.04141	-19.65	0.000	Significant
Sin(4pt)	-0.51956	0.04141	-12.55	0.000	Significant
Cos(4pt)	0.59914	0.04141	14.47	0.000	Significant
Sin(6pt)	0.27418	0.04141	6.62	0.000	Significant
Cos(6pt)	-0.23316	0.04141	-5.63	0.000	Significant
Sin(8pt)	-0.25587	0.04141	-6.18	0.000	Significant
Cos(8pt)	-0.11119	0.04141	-2.69	0.008	Significant
Sin(10pt)	0.12255	0.04141	2.96	0.003	Significant
Cos(10pt)	0.13646	0.04141	3.30	0.001	Significant

Table 4-10: ANOVA Table for Fourier 5 Sine and Cosine Pair Model for Alexis River

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Regression	324.043	10	32.404	112.49	0.00	significant
Error	93.618	325	0.288			
Total	417.661	335				

The regression model equation is shown below in Eqn. [26]. The assumptions of ANOVA, in which the residuals are normally distributed, independent and with constant variance, were met, as shown in Figure 4-36 below.

Ln Flow = 3.33 - 0.630 sin(2pt) - 0.814 cos(2pt) - 0.520 sin(4pt) + 0.599 cos(4pt) + 0.274 sin(6pt) - 0.233 cos(6pt) - 0.256 sin(8pt) - 0.111 cos(8pt) + 0.123 sin(10pt) + 0.136 cos(10pt) [26]

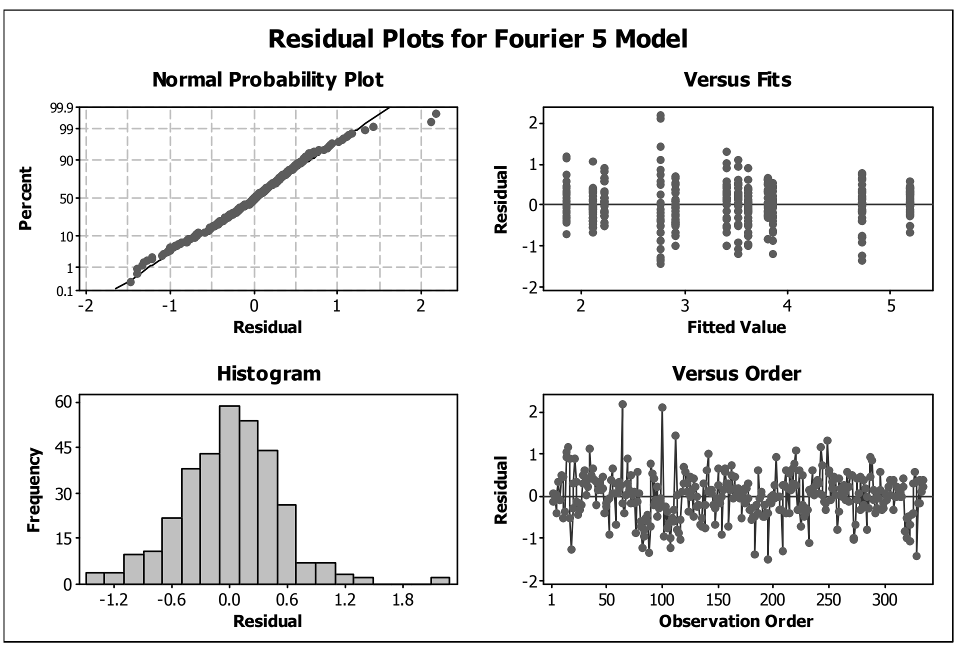


Figure 4-36: Fourier 5 Model Residual Tests for Alexis River

To verify the seasonal model, the fitted function values were compared to the natural log of the actual data for the record from 1978 to 2005. The natural log values were used for comparison since the data set is lognormal and the model was developed using lognormal flows. The model results, as well as forecast outputs, are produced as natural log flows. Figure 4-37 shows the Fourier fitted function models the lognormal actual values fairly well.

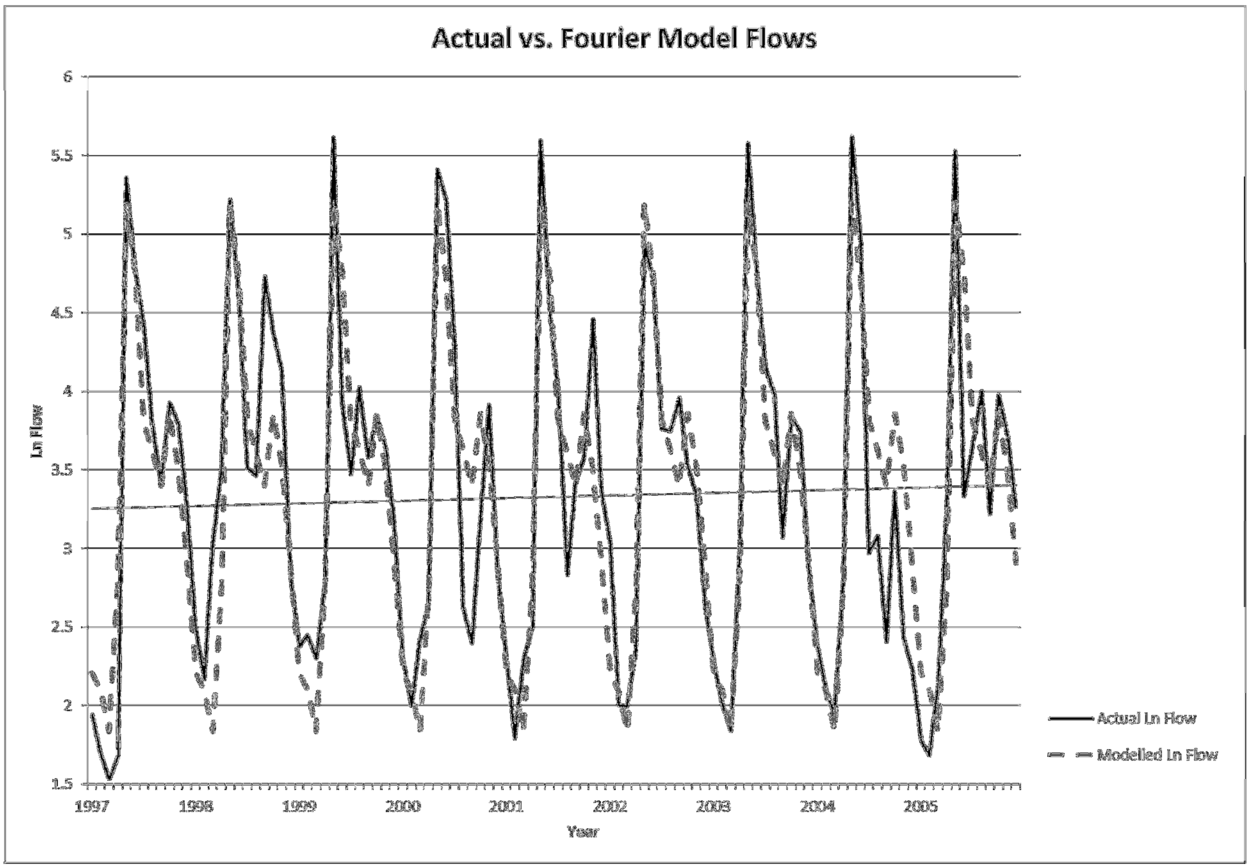


Figure 4-37: Actual Monthly Flows versus Fourier Model Values for Alexis River

The error between the fitted function values and the actual values was calculated using the NSE coefficient, MSD and MAPE. The closer the NSE coefficient is to 1 and the lower the MSD and MAPE values, the more closely the fitted function models the actual values. The error results are provided in Table 4-11.

Table 4-11: Calculated Error between Fourier Fitted Function and Actual Ln Flow Values for Alexis River

Years in data set	NSE Coefficient	MSD	MAPE
1978–2005	0.78	27.86	9.93%

Autocorrelation function and partial autocorrelation function plots were used to determine if there was any remaining seasonality in the residuals. In addition, the remaining lag correlation indicated the type of ARMA model that was required to address the autocorrelation. As shown in Figure 4-38 below, the seasonality was removed from the Fourier 5 model residuals, however there remained a lag 1 correlation and as such, further ARMA modeling was required to address the strong autocorrelation in the residuals.

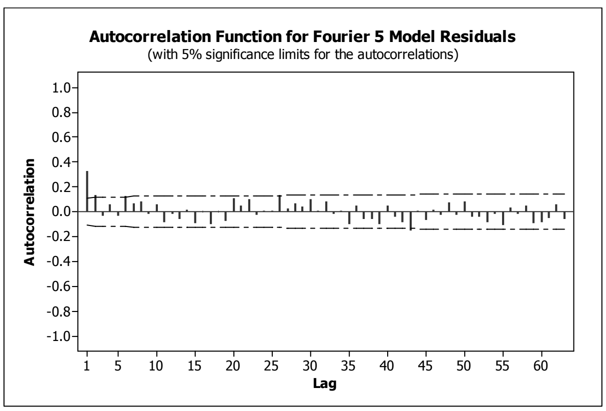


Figure 4-38: ACF Plot of Fourier Model Residuals for Alexis River

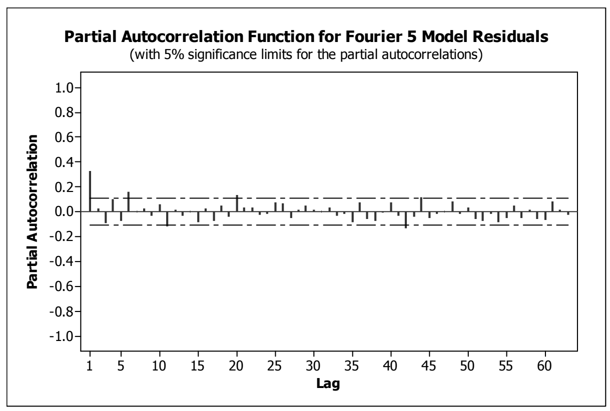


Figure 4-39: PACF Plot of Fourier Model Residuals for Alexis River

4.2.1.2 ARMA Analysis of Deseasonalized Flows

Since the seasonality was removed from the Alexis River monthly data set using Spectral Analysis and Fourier Modeling, the remaining deseasonalized data (residuals) was modeled using ARMA. As previously shown in the ACF and PACF plots of the residuals in Figure 4-38 and Figure 4-39, there was a lag 1 correlation in the PACF plot which suggests an AR1 model might be most appropriate (Box et al, 2008) For completeness, a number of different ARMA models were developed and the statistics of each reviewed to determine the best fitting ARMA model for the Alexis actual data set. Table 4-12 summarizes the results of the reviewed ARMA models complete with corresponding ACF and PACF plots of the model residuals.

Table 4-12: Results of ARMA models for Alexis River Deseasonalized Flows

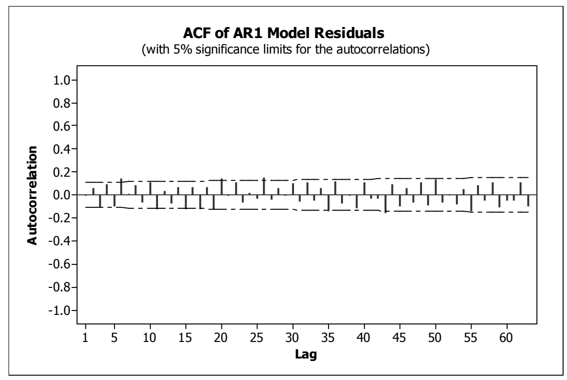
Model Type	Model Results	ACF and PACF Plots
AR1		

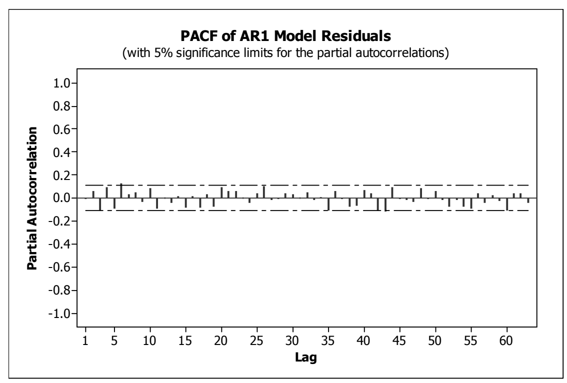
Type	Coef	SE Coef	T	P
AR 1	0.3283	0.0517	6.35	0.000
Constant	0.00042	0.02728	0.02	0.988
Mean	0.00062	0.04062		

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	32.33	68.6	104.7	142.9
DF	10	22	34	46
P-Value	0.000	0.000	0.000	0.000

AR1 coefficient is significant and there is a significant lag 6 correlation remaining in the ACF and PACF plots. The Ljung-Box tests are significant suggesting the data is not independent or random. This model was compared to other models with significant coefficients.





Model Type	Model Results	ACF and PACF Plots

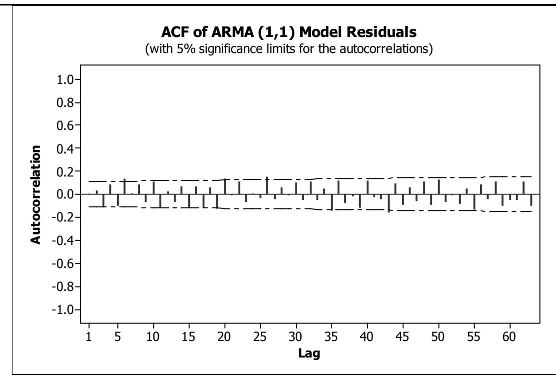
ARMA (1,1)

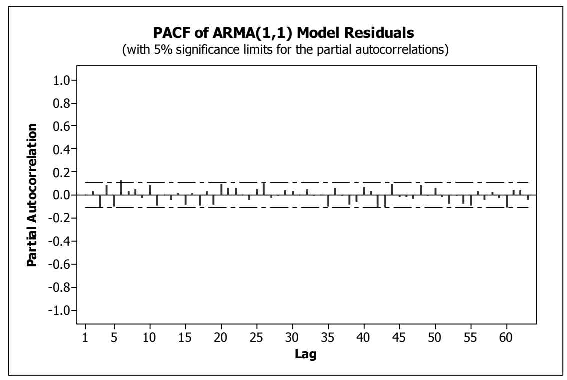
Type	Coef	SE Coef	T	P
AR 1	0.3788	0.1541	2.46	0.014
MA 1	0.0564	0.1662	0.34	0.735
Constant	0.00041	0.02578	0.02	0.987
Mean	0.00067	0.04150		

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	30.6	65.0	99.5	136.2
DF	9	21	33	45
P-Value	0.000	0.000	0.000	0.000

The MA1 coefficient is not significant therefore this model is not a good fit for the actual data.



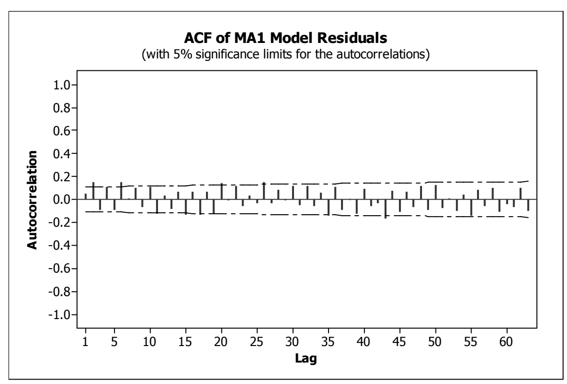


MA 1

Type	Coef	SE Coef	T	P
MA 1	-0.2686	0.0527	-5.09	0.000
Constant	0.00034	0.03496	0.01	0.992
Mean	0.00034	0.03496		

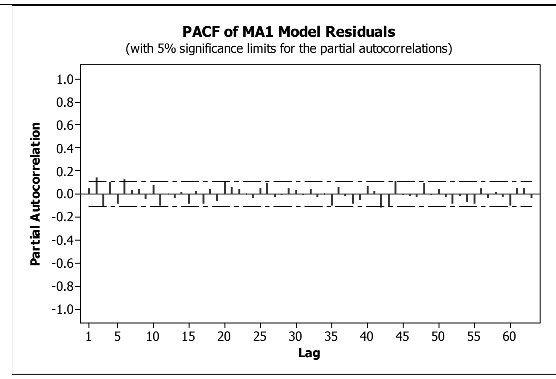
Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	40.6	80.0	117.4	157.9
DF	10	22	34	46
P-Value	0.000	0.000	0.000	0.000



Model Type	Model Results	ACF and PACF Plots

MA1 coefficient is significant however all four of the Ljung-Box tests are significant suggesting the data are not independent or random. There are significant lag 2 and lag 6 correlations remaining. This model will be compared to other models with significant coefficients.



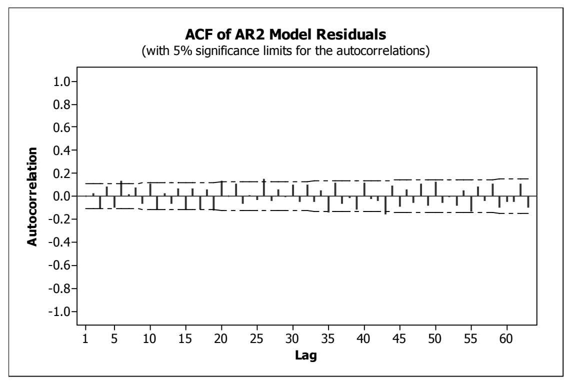
AR 2

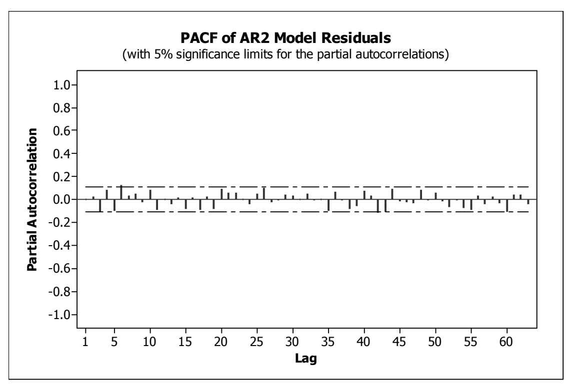
Type	Coef	SE Coef	T	P
AR 1	0.3191	0.0548	5.82	0.000
AR 2	0.0280	0.0548	0.51	0.610
Constant	0.00045	0.02732	0.02	0.987
Mean	0.00069	0.04184		

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	29.6	62.9	96.5	132.4
DF	9	21	33	45
P-Value	0.001	0.000	0.000	0.000

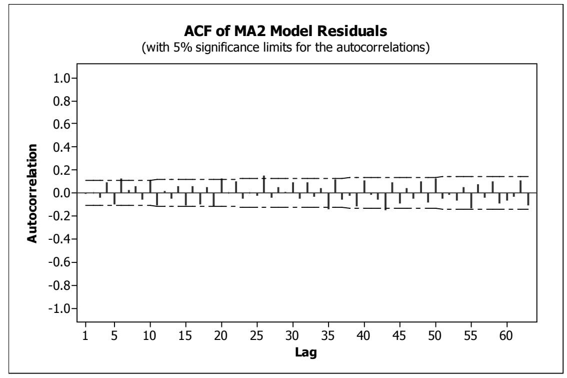
The AR1 coefficient is significant but the AR2 coefficient is not; therefore this model is not a good fit for the actual data.





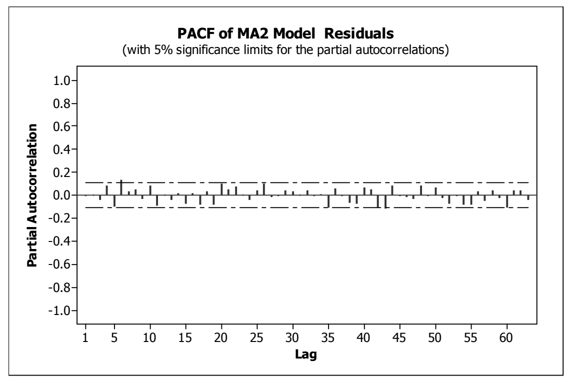
MA2

Type	Coef	SE Coef	T	P
MA 1	-0.3378	0.0541	-6.24	0.000
MA 2	-0.1591	0.0541	-2.94	0.004
Constant	0.00055	0.04071	0.01	0.989
Mean	0.00055	0.04071		



Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	23.9	51.5	82.0	114.1
DF	9	21	33	45
P-Value	0.000	0.000	0.000	0.000



Both MA1 and MA2 coefficients are significant however all of the Ljung Box statistics are significant suggesting the data are not random or independent. There is a significant lag 6 correlations remaining. This model will be compared to other models with significant coefficients.

Errors in model development were encountered using Minitab and models could not be fully generated for ARMA (2,1), ARMA (1,2) and ARMA (2,2) so these were not examined further. AR2 and ARMA (1,1) did not have all coefficients as significant so these were not considered to be good models of the deseasonalized data. AR1, MA1 and MA2 all had coefficients as significant, however, the Ljung-Box Chi Square statistics were not significant indicating the deseasonalized data were neither independent nor random, which would be expected for monthly flow data. A review of the ACF and PACF plots for these three ARMA models, AR1, MA1 and MA2, indicated that all had a significant lag 6 correlation remaining and MA1 also had a significant lag 2 correlation.

With the ACF and PACF plots being similar between these three models, the principle of parsimony was applied and the simplest model was selected. In this case, the AR1 model was selected as best fitting since it effectively addressed the significant lag correlations, with the exception of lag 6, and was the simplest to understand.

The AR1 model results, as shown above in Table 4-12, indicate that the AR1 parameter was significant. The ACF and PACF plots of the AR1 model residuals, also shown in Table 4-12, indicate that all but the lag 6 correlations have been addressed and thus the AR1 model is good at modeling the residuals of the Alexis River deseasonalized monthly data. The assumptions of ARMA in which normally distributed residuals and with constant variance and independence, are met as shown in Figure 4-40 below.

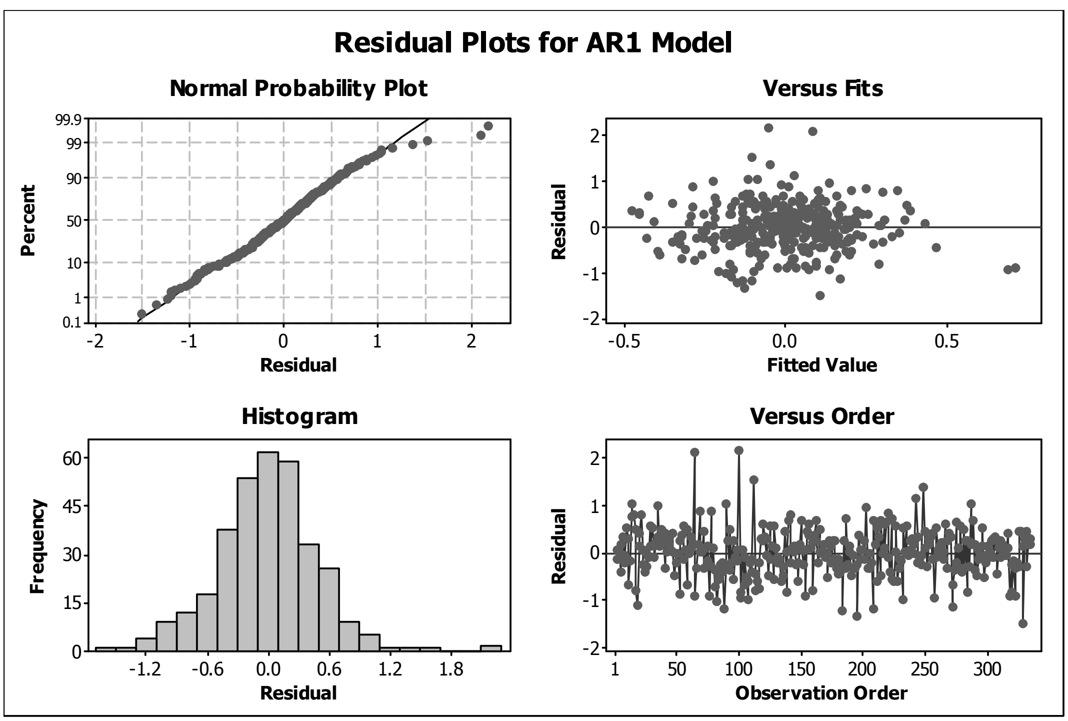


Figure 4-40: AR1 Model Residual Tests for Alexis River

Simulation was used to verify that the data set could indeed generate a data set of the same sample size with similar statistics. To verify the AR1 model, the model was required to reproduce the statistics of the actual data, which in this case are the residuals after the data set was deseasonalized. A Monte Carlo simulation of an AR1 process was completed whereby five hundred replications were used to determine if, on average, the AR1 model could reproduce the historical statistics. Below, in Table 4-13, is a comparison of the statistics for the actual deseasonalized data versus the AR1 simulated values.

Table 4-13: Comparison of AR1 Simulated Statistics to Statistics for Log-transformed Alexis River Deasonalized Residuals

	N	Mean	Std dev	Min Flow	Max Flow	Skew	r1
Actual	336	0.00	0.53	-1.47	2.19	0.13	0.3278
Simulated	336	-0.002	0.53	-1.54	1.54	0.003	0.3179
LCL		-0.08	0.49	-1.93	1.12	-0.26	0.2191
UCL		0.08	0.57	-1.16	1.96	0.27	0.4167

An acceptable AR1 model should be able to reproduce the mean, standard deviation and lag one correlation on average to within the 95% confidence limits. As shown in Table 4-13 above, the mean, standard deviation and lag one correlation (r1) were within the 95% confidence interval. The other actual statistics should also be within the 95% confidence interval of the simulated statistics, however, for this model only the minimum flow and the skew were within the limits while the maximum flow was not. Since these values were not explicitly modeled in the simulation, they would be difficult to preserve.

To graphically compare the actual data and the simulated data, time series plots for both were completed and are in Figure 4-41 and Figure 4-42 below. As shown, the time series plot for the simulated data looks similar to the time series plot for the actual data set, and the AR1 model therefore appears to be a good model representation of the deseasonalized data of the Alexis River.

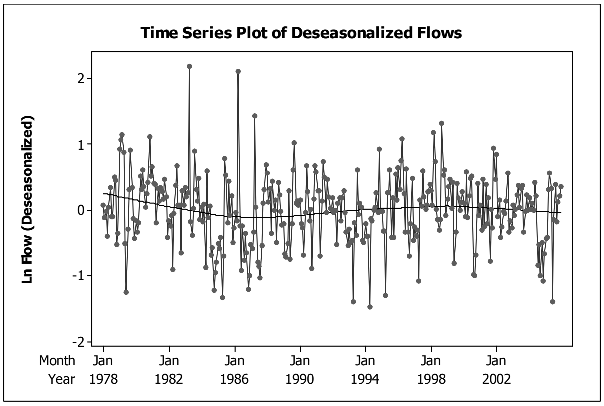


Figure 4-41: Time Series Plot of Deseasonalized Actual Data for Alexis River

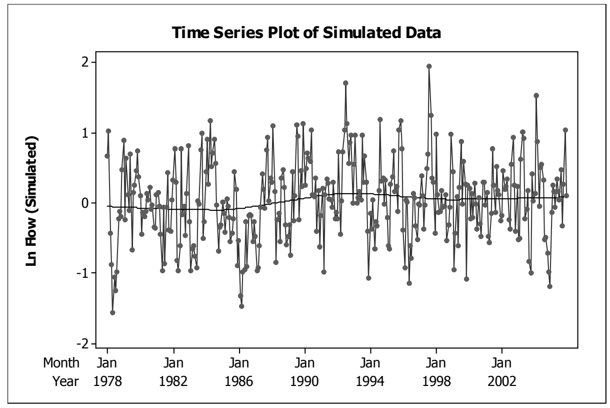


Figure 4-42: Time Series Plot of One AR1 Simulated Data Set for Alexis River

4.2.1.3 Flow Simulations

Estimating energy production and reliability for prospective hydroelectric developments requires simulation of streamflows over a long term. These estimates can be made using a flow duration curve. Flow duration curves have traditionally been constructed deterministically using a single existing hydrometric record. Multiple simulations based on time series analysis however, can capture the expected streamflow variability over time. Since the Alexis River has potential as a hydroelectric development site, multiple simulations of flows can be used to estimate its energy potential. The simulations can be used to construct a set of flow duration curves for the river that can be used for capacity planning as well as reliability analysis. If a reservoir is planned for the site, the simulations can provide multiple inflow sequences to a reservoir model.

The following curves for the Alexis River were developed using the Fourier 5 model results in conjunction with nine AR1 model simulations for 28 years, from 1978-2005 inclusive. These flow duration curves are shown in Figure 4-43 and together illustrate a range of simulated curves for the Alexis River. For comparison, the flow duration curve based on the actual measured flows, from 1978-2005, is in Figure 4-44 and shows similarities to the flow duration curves developed using multiple simulations.

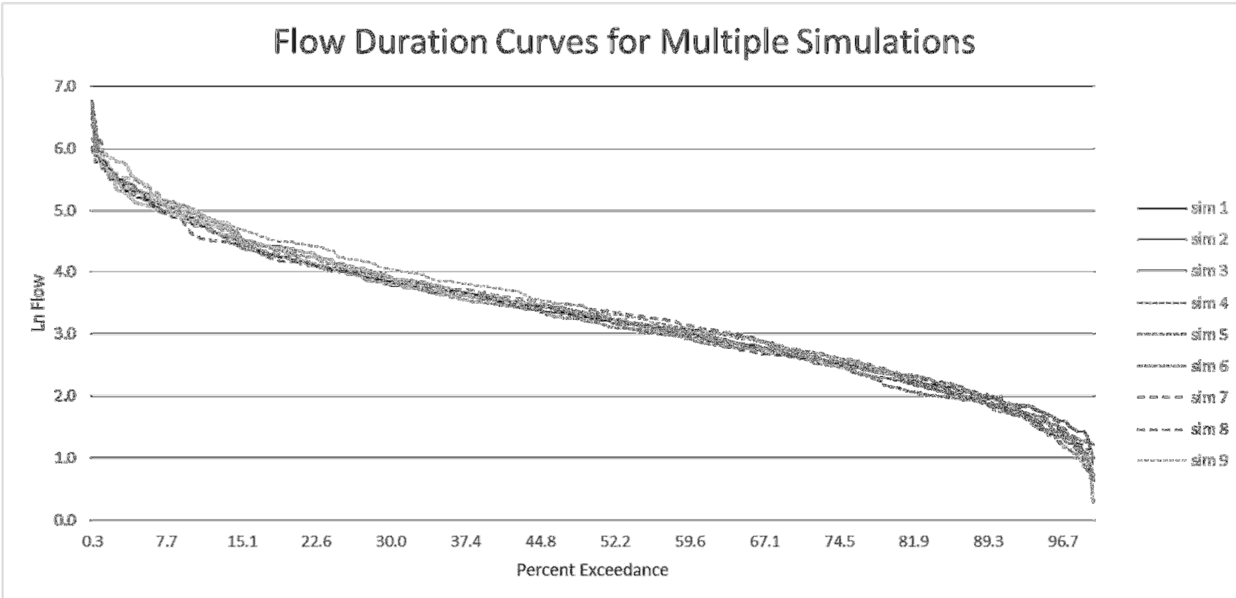


Figure 4-43: Simulated Flow Duration Curves for Alexis River

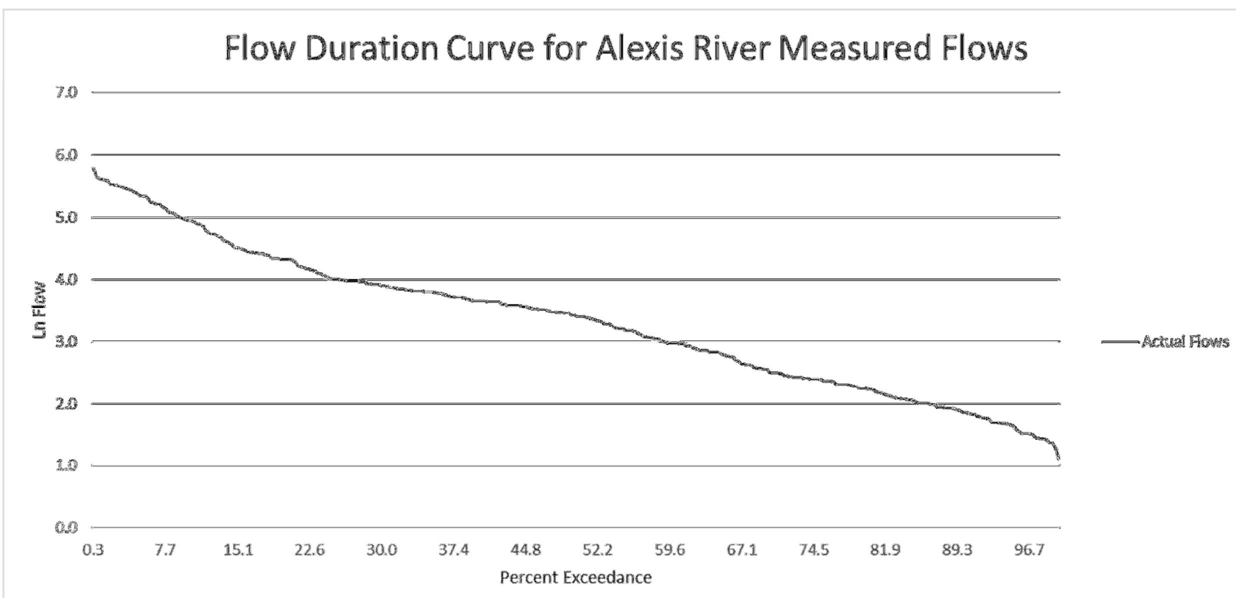


Figure 4-44: Flow Duration Curve for Ln Actual Alexis River Flows

4.2.1.4 Forecasting

The Fourier model was used to forecast five years of flow values, from 2006 to 2010. These correspond to the years that were removed from the original data set prior to model development. These flows were removed from the model data set so that the forecast accuracy of the model could be calculated and compared against the actual flow data from the same time period. A period of five years was selected to forecast even though it is well understood that, in practise, forecasts more than 12 months beyond the end of the model period lose relevance (Sidhu and Lye, 1995).

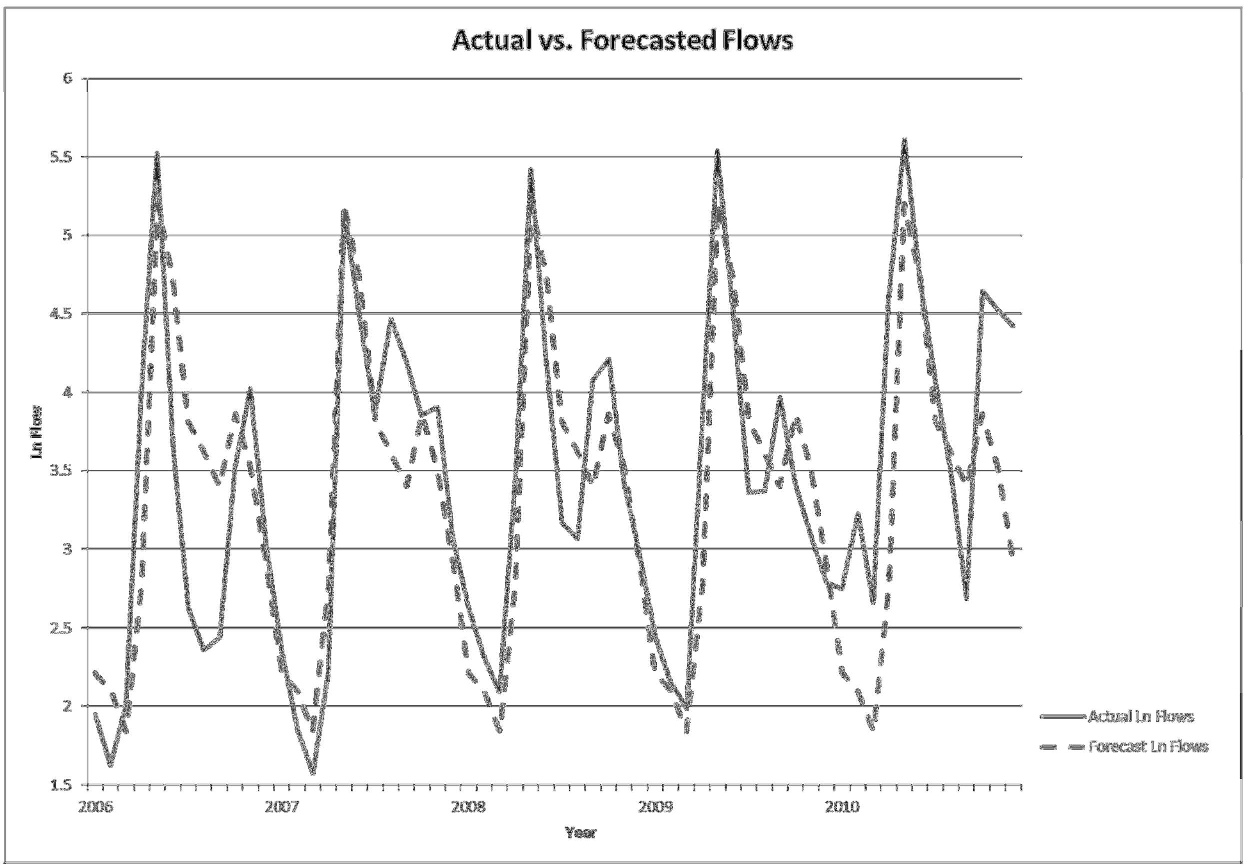


Figure 4-45: Actual vs Forecast Flows for Alexis (2006-2010)

To validate the prediction ability of the model, the predicted values for the last five years of data were graphically compared to the actual last five years of data, from 2006 to 2010. As shown in Figure 4-45, the model appears to do a good job of predicting the first four years (2006 to 2009) but does not fit the actual data well for from Nov/Dec 2009 to March 2010 as well as July to Dec 2010. A closer look at the actual data suggests that the peak patterns that exist in the data pre-2006 do not appear to be replicated between the end of 2009 and end of 2010, even though the pattern is replicated between 2006 and 2009.

Three methods were used to validate the predictions: Nash–Sutcliffe efficiency, Mean Squared Deviation and Median Absolute Percentage Error. Since the actual data for late 2009–2010 did not appear to have a pattern consistent with previous years' data, the data set was segregated to specifically look at the data with and without the late 2009 and 2010 years. Table 4-14 shows the error calculation results.

Table 4-14: Validation Results for ARMA with Spectral Analysis Forecasts of Alexis River

Years in set reviewed	**NSE coefficient**	**MSD**	**MAPE**
Full Set (1978–2010)	0.76	29.84	10.25%
2006–2010	0.63	40.88	12.76%
2006–2008	0.70	33.46	12.58%
2009–2010	0.52	52.01	13.13%

As shown in the table, the predicted flows between 2006 and 2010 show a value of 0.63 which is less efficient than the full data set at 0.76. As noted above, a graphical review of Figure 4-45 indicates that actual flows between the winter of 2009 and winter 2010 did not appear to display the same pattern as historical flows had shown; primarily there were higher actual flows during the winter period than observed historically. As a result, the Nash–Sutcliffe efficiency coefficient was reviewed separately for the 2006-2008 flows and the 2009-2010 flows. As noted in the table above, the 2006–2008 predicted flows better match the actual data than do 2006-2010 or 2009-2010.

Similar results were found when the MAPE was reviewed for the same sets of flows. The table shows that there is a larger percentage error (13.13%) between the predicted and actual flows for 2009-2010 and a smaller percentage error (12.58%) between predicted and actual flows for 2006–2008. A review of the

MSD also indicates that the 2006-2008 is a better fit to the actual flows than the other years. Given the historical pattern in the actual data was not replicated in 2009 and 2010, this difference would not be represented by the model. It would be difficult to predict this change in pattern and it would be expected that the model would not forecast these years well. Based on these results, this selected model, generated using deseaonalized ARMA, was used to predict short term flows with reasonable accuracy. The larger discrepancy between measured and predicted flows in 2009 and 2010 appears to result from a different distribution of seasonal flows where the flows are higher in the winter for these two years. This might suggest a warmer winter and/or earlier snow melt.

4.2.1.5 Summary

Analysis of the monthly flows from the Alexis River in Southern Labrador shows no trend but did show seasonality in the data set with three periods; 12 months, 6 month and 4 months. As is typical for hydrologic data sets, the data required a log transformation to improve normality and as such, the models were developed on the transformed data set. A time series analysis approach was selected to develop a model of the Alexis River monthly data. In order to develop a comprehensive model of the flow, a combination approach was selected; model the seasonality using frequency domain spectral analysis and model the remaining residuals using the time domain ARMA model. A Fourier 5 sine and cosine pair model was selected to address the periodicity as it best addressed the seasonality in the data. An AR1 model was selected to model the remaining deseasonalized data as it best addressed the correlation found in the residuals. The AR1 model was able to reproduce the time series plot and the actual statistics within accepted tolerances. The model was used to forecast short term flows with reasonable accuracy based on NSE, MSD and MAPE error calculations as well as construct a range of flow duration curves.

4.2.2 Ugjoktok River

Preliminary analysis of the Ugjoktok River streamflows indicated that the data was not normally distributed as evidenced by a normality test and box plot, however, the data were found to be normal after a log transformation. Box plots showed seasonality in the data and there was no significant trend detected in the fitted line test. Tests for autocorrelation revealed there is short term dependence in the flow data for the Ugjoktok River. The data from 1979 to 2005 were used to develop the model, with the last five years of the actual data set, 2006 to 2010, being used to compare the model forecast values to actual values.

4.2.2.1 Seasonal Analysis

Spectral Analysis was used to identify the periods associated with periodic data sets and model the seasonality. Since the monthly Ugjoktok River data have been shown to have periodicity, the identification of the periods was accomplished using spectral analysis. A scatterplot of the spectral density function versus period is shown below in Figure 4-46 and of the spectral density function versus frequency is shown in Figure 4-47. These plots, in conjunction with the results in Minitab, indicated there were three frequencies with the UCL to LCL range being significant: one at 0.08, one at 0.17, and one at 0.25. These noted frequencies correspond to periods of 12 months, 6 months and 4 months and are the same frequencies identified in the spectral analysis of Alexis River.

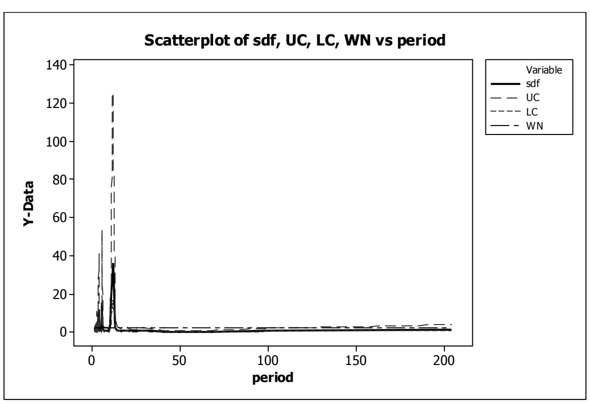


Figure 4-46: Scatterplot of Spectral Density Function versus Period for Ugjoktok River Data

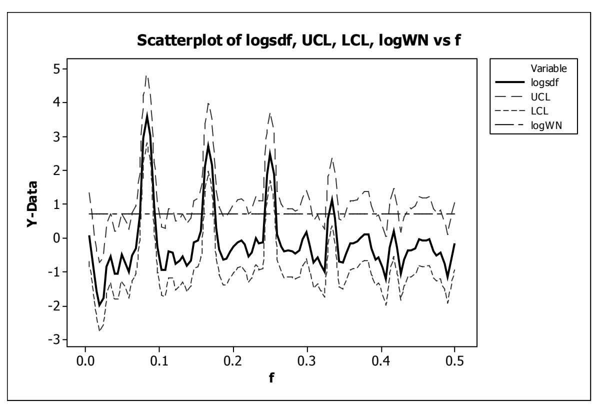


Figure 4-47: Scatterplot of Spectral Density Function versus Frequency for Ugjoktok River Data

With seasonality and the periods confirmed by spectral analysis, the seasonal component of the data set was modeled using the Fourier method. Analysis proved that the best fitting equation included 5 sine and cosine pairs, where all the pairs are significant, the residual error is low and adjusted R^2 value is the highest of all the models at 86.7%. As a result, the Fourier 5 pair model was selected. The results of the regression analysis are shown below in Table 4-15 and Table 4-16. The predictors listed in the table form part of the regression equation and in addition to a constant value, include sine and cosine pairs, indicating the ultimate number of pairs in the regression equation.

Table 4-15: Regression Results of Fourier 5 Sine and Cosine Pair Model for Ugjoktok River

Predictor	**Coefficient**	**SE Coefficient**	**T**	**P**	
Constant	4.41115	0.02473	178.38	0.000	Significant
Sin(2pt)	-1.08376	0.03497	-30.99	0.000	Significant
Cos(2pt)	-0.82675	0.03497	-23.64	0.000	Significant
Sin(4pt)	-0.15186	0.03497	-4.34	0.000	Significant
Cos(4pt)	0.64755	0.03497	18.52	0.000	Significant
Sin(6pt)	0.15802	0.03497	4.52	0.000	Significant
Cos(6pt)	-0.45033	0.03497	-12.88	0.000	significant
Sin(8pt)	-0.15719	0.03497	-4.49	0.000	significant
Cos(8pt)	0.09198	0.03497	2.63	0.009	significant
Sin(10pt)	0.13853	0.03497	3.96	0.000	significant
Cos(10pt)	0.05614	0.03497	1.61	0.109	Not significant

Table 4-16: ANOVA Table of Fourier 5 Sine and Cosine Pair Model for Ugjoktok River

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Regression	418.564	10	41.856	211.26	0.00	significant
Error	62.014	313	0.198			
Total	480.578	323				

The regression model equation is as shown below in Eqn. [27]. The assumptions of ANOVA, normally distributed residuals with constant variance and independence, are met, as shown in Figure 4-48 below.

$$\mathbf{Ln\ Flow} = \mathbf{4.41 - 1.08\sin(2pt) - 0.827\cos(2pt) - 0.152\sin(4pt) + 0.648\cos(4pt) + 0.158\sin(6pt) - 0.450\cos(6pt) - 0.157\sin(8pt) + 0.0920\cos(8pt) + 0.139\sin(10pt) + 0.0561\cos(10pt)} \tag{27}$$

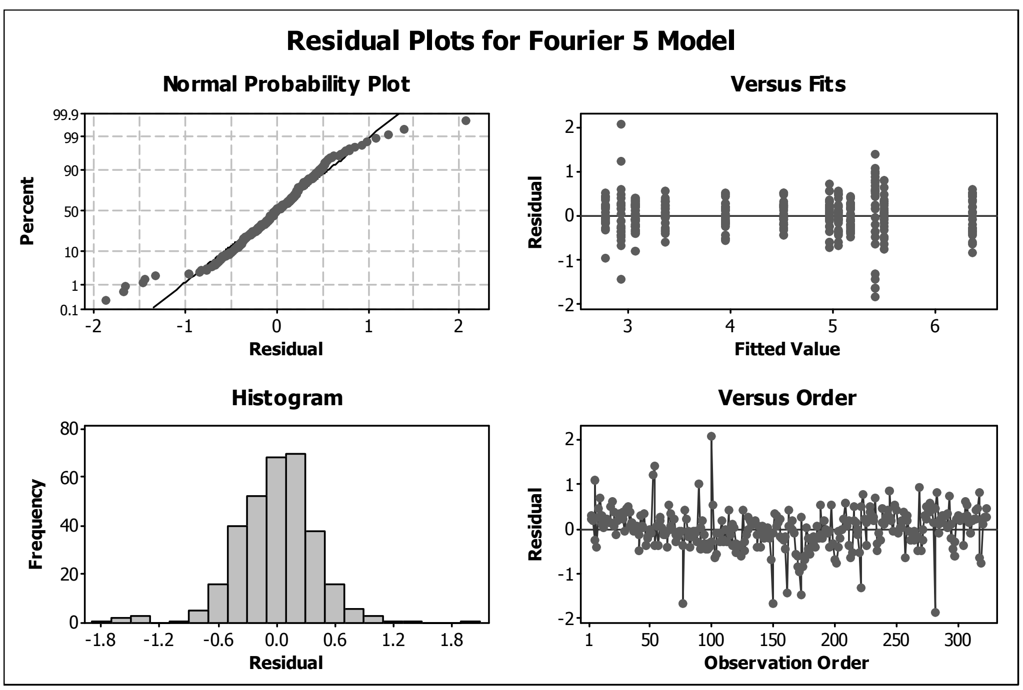


Figure 4-48: Fourier 5 Model Residual Tests for Ugjoktok River

In order to verify the seasonal model, the fitted function were compared to the actual data for the record from 1979 to 2005. As shown in Figure 4-49 below, the fitted function models the actual values fairly well.

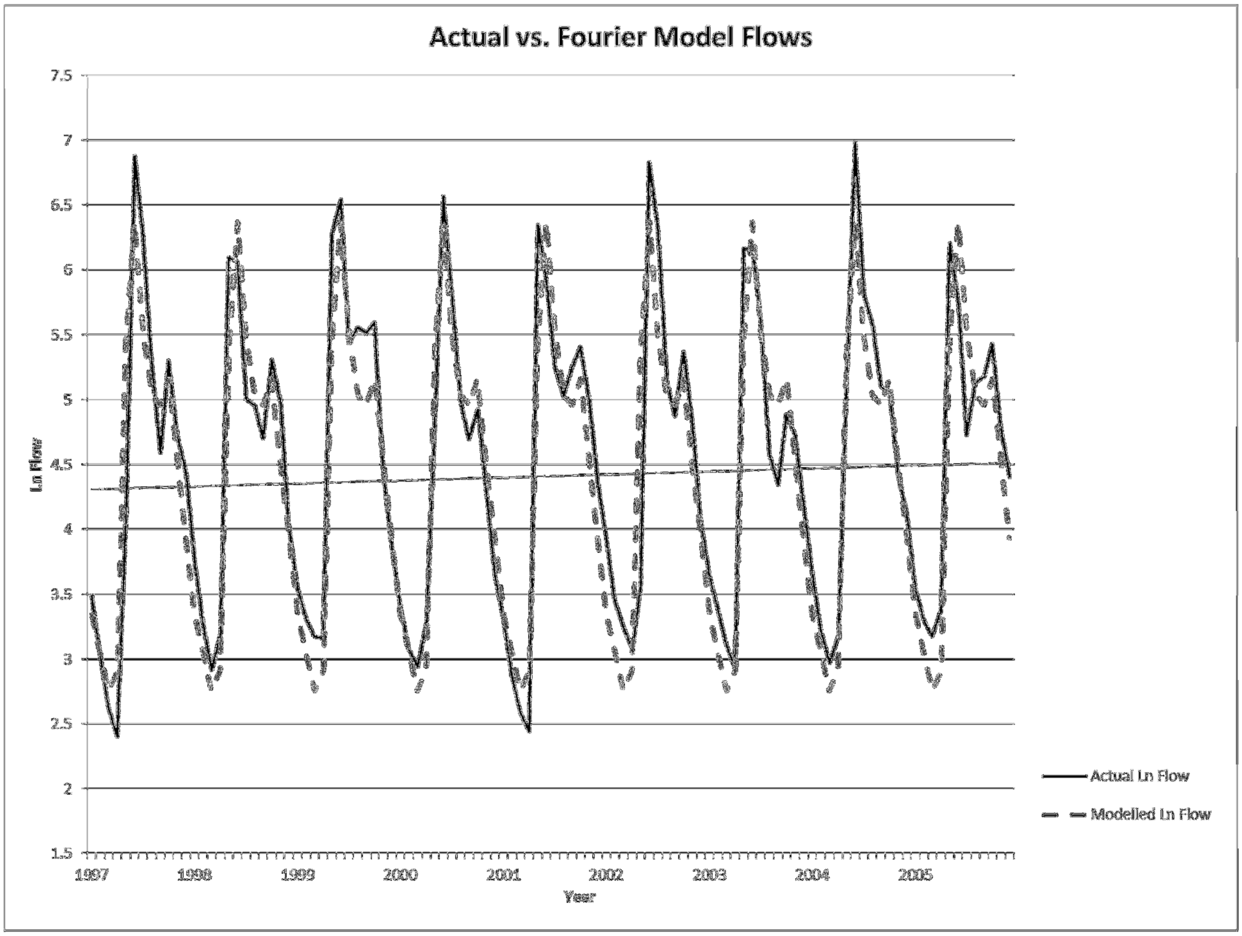


Figure 4-49: Actual Monthly Flows versus Fourier Model Values for Ugjoktok River

The error between the fitted function values and the actual values was calculated using the NSE coefficient, MSD and MAPE. The error results are provided in Table 4-17.

Table 4-17: Calculated Error between Fourier Fitted Function and Actual Flow Values for Ugjoktok River

Years in data set	NSE Coefficient	MSD	MAPE
1979–2005	0.87	19.14	5.77%

ACF and PACF plots were completed to determine if there was any remaining seasonality in the residuals. In addition, the remaining lag correlation indicated the type of ARMA model that was required to address the autocorrelation. As shown in Figure 4-50 and Figure 4-51 below, the seasonality was removed from the Fourier 5 model residuals, however there remained a lag 1 correlation and as such, further ARMA modeling was required to address the strong autocorrelation in the residuals.

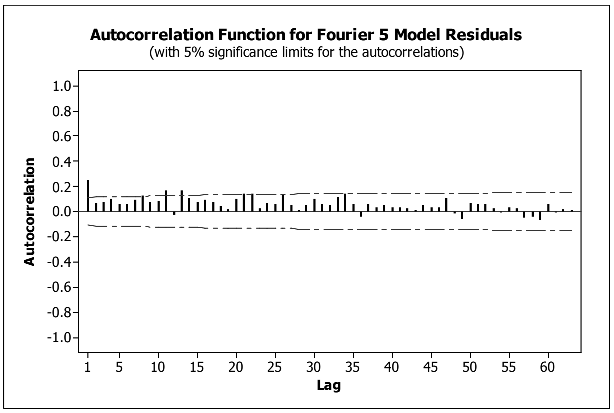


Figure 4-50: ACF Plot of Fourier Model Residuals for Ugjoktok River

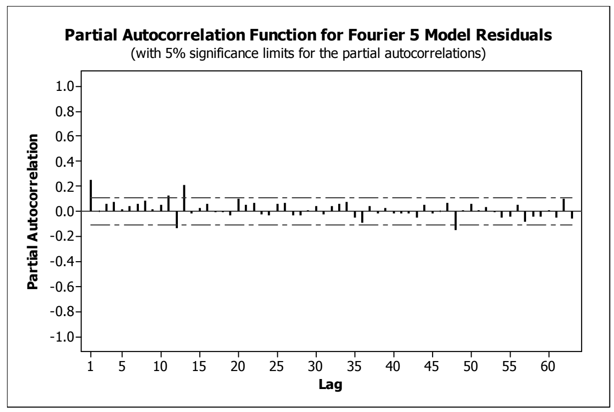


Figure 4-51: PACF Plot of Fourier Model Residuals for Ugjoktok River

4.2.2.2 ARMA Analysis of Deseasonalized Flows

With the seasonality removed from the Ugjoktok River monthly data set using Spectral Analysis and Fourier Modeling, the remaining residuals were modeled using ARMA. As previously shown in the ACF and PACF of the residuals, Figure 4-50 and Figure 4-51, there is a lag 1 correlation in the PACF plot which suggests an AR1 model might be most appropriate. Similar to the ARMA analysis for Alexis River, different ARMA models were completed and the statistics of each were reviewed to determine the best fitting ARMA model for the Ugjoktok actual data set. Table 4-18 summarizes the results of the reviewed ARMA models along with corresponding ACF and PACF plots of the model residuals. .

Table 4-18: Results of Different ARMA Models for Ugjoktok River

Model Type	**Model Results**	**ACF and PACF Plots**

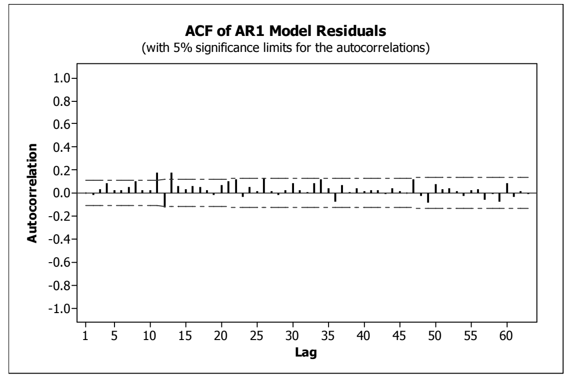
AR1

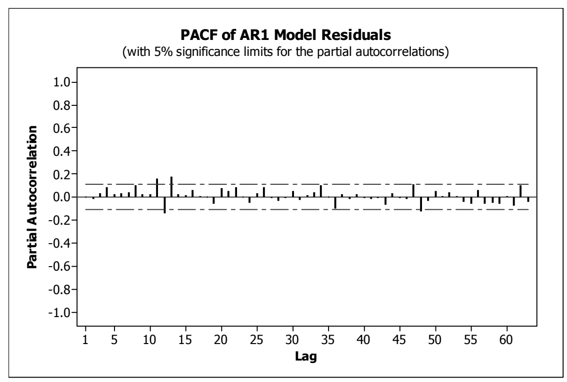
Type	Coef	SE Coef	T	P
AR 1	0.2556	0.0540	4.74	0.000
Constant	0.00062	0.02357	0.03	0.979
Mean	0.00084	0.03167		

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	23.2	47.7	66.2	75.8
DF	10	22	34	46
P-Value	0.010	0.001	0.001	0.004

AR1 coefficient is significant however there are significant lag correlations remaining around lags 11 and 13 and the Ljung-Box tests are significant suggesting the data is not independent or random. This model will be compared to other models with significant coefficients.





ARMA (1,1)

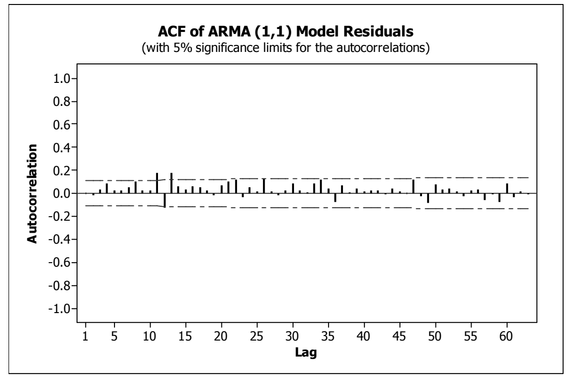
Type	Coef	SE Coef	T	P
AR 1	0.2533	0.2117	1.20	0.233
MA 1	-0.0025	0.2188	-0.01	0.991
Constant	0.00062	0.02367	0.03	0.979
Mean	0.00083	0.03170		

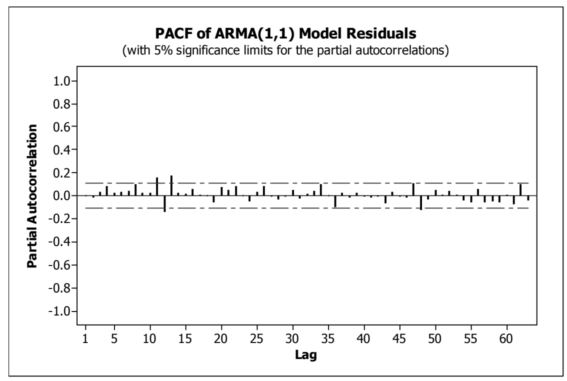
Model Type	Model Results	ACF and PACF Plots

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	23.2	47.8	66.3	75.8
DF	9	21	33	45
P-Value	0.006	0.001	0.001	0.003

Both AR1 and MA1 coefficients are not significant therefore this model is not a good fit for the actual data.



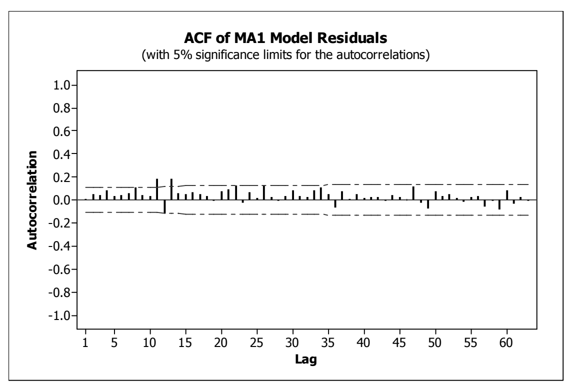


MA 1

Type	Coef	SE Coef	T	P
MA 1	-0.2470	0.0541	-4.57	0.000
Constant	0.00053	0.02944	0.02	0.986
Mean	0.00053	0.02944		

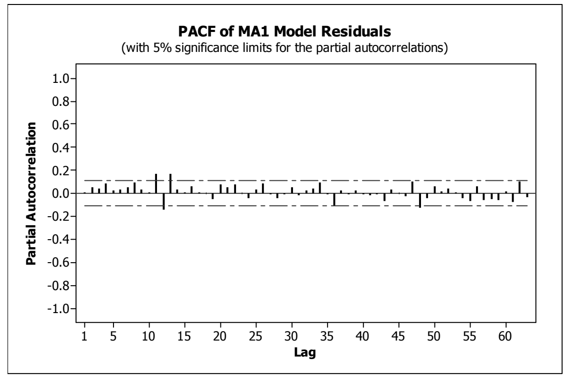
Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	27.4	55.4	71.7	87.4
DF	10	22	34	46
P-Value	0.002	0.000	0.000	0.000



Model Type	Model Results	ACF and PACF Plots

MA1 coefficient is significant however all four of the Ljung-Box tests are significant suggesting the data are not independent or random. There are significant lag correlations remaining around lags 11 and 13. This model will be compared to other models with significant coefficients.



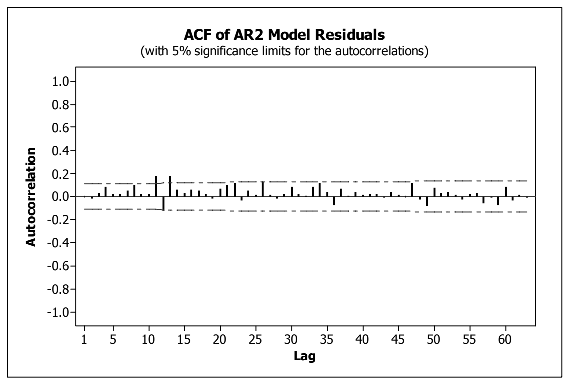
AR 2

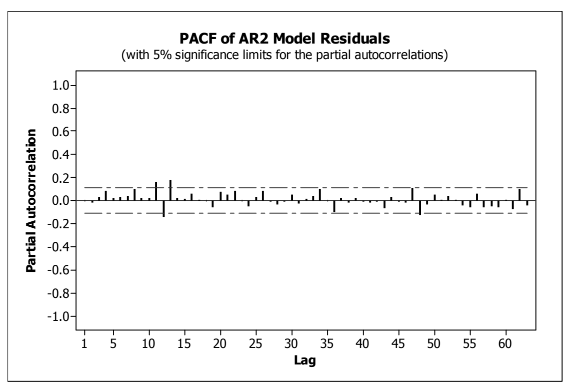
Type	Coef	SE Coef	T	P
AR 1	0.2557	0.0559	4.58	0.000
AR 2	-0.0003	0.0559	-0.01	0.995
Constant	0.000062	0.02361	0.03	0.979
Mean	0.00083	0.03171		

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	23.2	47.7	66.2	75.8
DF	9	21	33	45
P-Value	0.006	0.001	0.001	0.003

The AR1 coefficient is significant but the AR2 coefficient is not, therefore this model is not a good fit for the actual data.

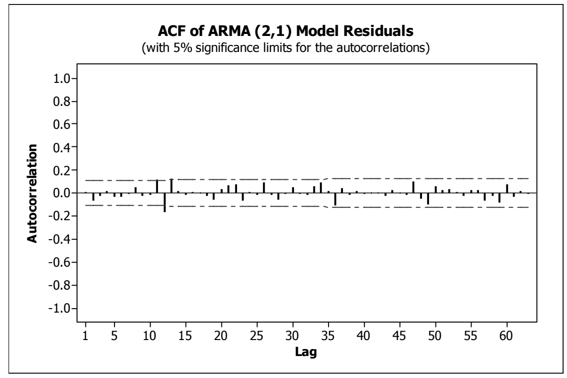




Model Type	Model Results	ACF and PACF Plots

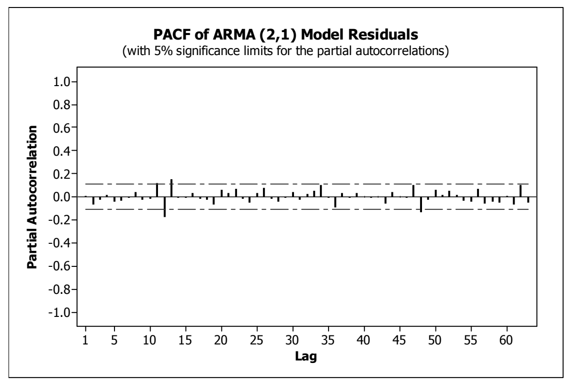
ARMA 2,1

Type	Coef	SE Coef	T	P
AR 1	1.1677	0.0602	19.41	0.000
AR 2	-0.1760	0.0557	-3.16	0.002
MA 1	0.9527	0.0312	30.56	0.000
Constant	0.000307	0.001153	0.27	0.790
Mean	0.0367	0.1381		



Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	17.8	30.0	43.9	50.2
DF	8	20	32	44
P-Value	0.023	0.069	0.078	0.242



ARMA 2,1 coefficients are significant however Minitab was unable to reduce the sum of squares which suggests this model does not fit the data set well. Three of the Ljung-Box tests were not significant indicating the data is independent and random.

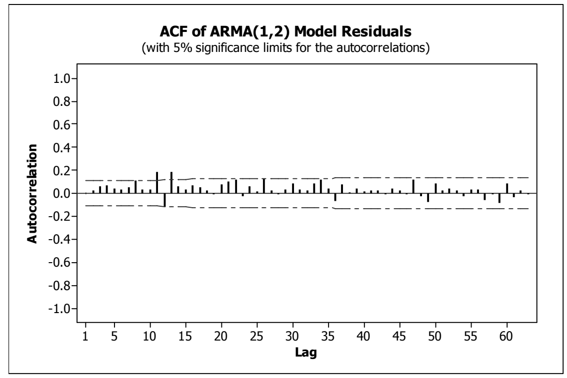
ARMA 1,2

Type	Coef	SE Coef	T	P
AR 1	-0.5602	1.1301	-0.50	0.620
MA 1	-0.8162	1.1235	-0.73	0.468
MA 2	-0.1729	0.2597	-0.67	0.506
Constant	0.00083	0.04706	0.02	0.986
Mean	0.00053	0.03016		

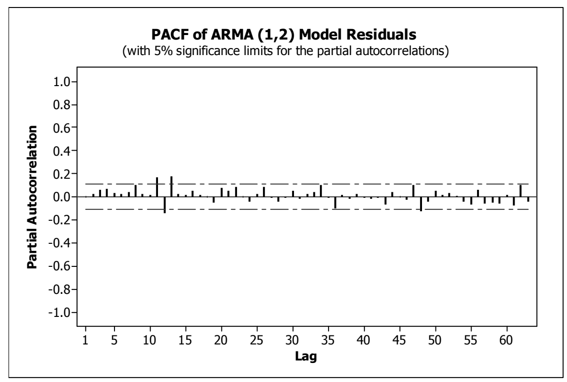
Model Type	Model Results	ACF and PACF Plots

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	25.8	53.0	71.7	81.9
DF	8	20	32	44
P-Value	0.001	0.000	0.000	0.000

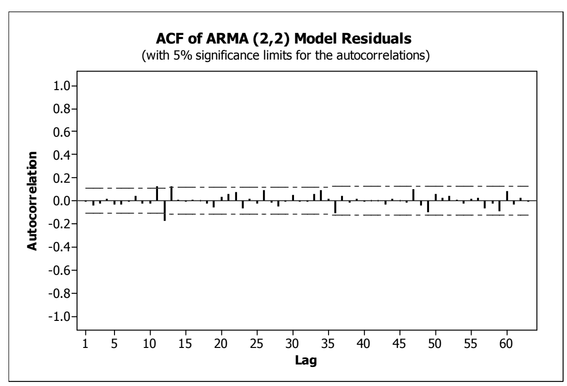


The AR1 and MA1 coefficients not are significant, therefore this model is not a good fit for the actual data.



ARMA 2,2

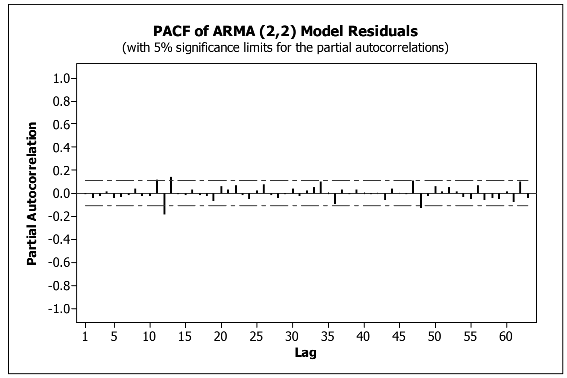
Type	Coef	SE Coef	T	P
AR 1	1.0519	0.3706	2.84	0.005
AR 2	-0.0617	0.2564	-0.24	0.810
MA 1	0.8109	0.3704	2.19	0.029
MA 2	0.1343	0.1862	0.72	0.471
Constant	0.000399	0.001560	0.26	0.798
Mean	0.0407	0.1589		



Model Type	Model Results	ACF and PACF Plots

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

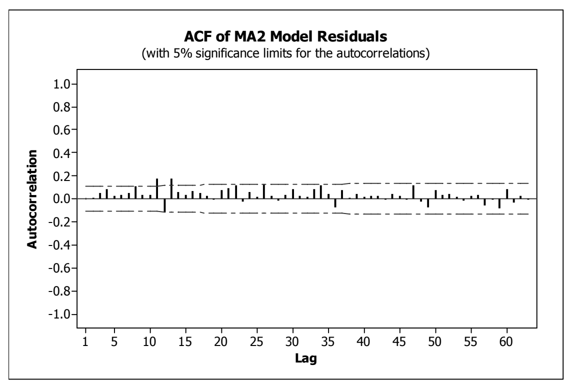
Lag	12	24	36	48
Chi-Square	19.8	31.5	42.9	55.3
DF	7	19	31	43
P-Value	0.006	0.035	0.076	0.099



The AR1 and MA1 coefficients are significant however the AR2 and MA2 coefficients are not, therefore this model is not a good fit for the actual data.

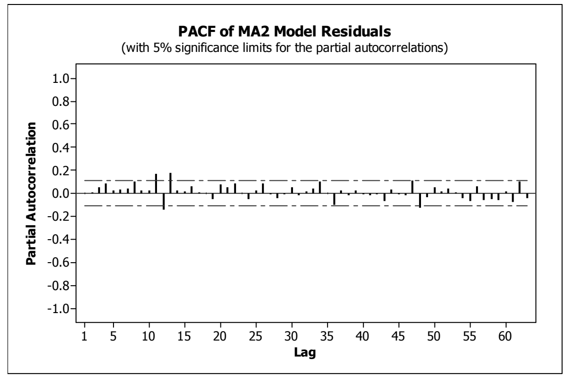
MA2

Type	Coef	SE Coef	T	P
MA 1	-0.2564	0.0558	-4.59	0.000
MA 2	-0.0414	0.0559	-0.74	0.459
Constant	0.00066	0.03066	0.02	0.983
Mean	0.00066	0.03066		



Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	24.7	50.9	69.6	79.5
DF	9	21	33	45
P-Value	0.003	0.000	0.000	0.000

Model Type	Model Results	ACF and PACF Plots
	MA1 coefficient is significant however the MA2 coefficient is not significant and therefore this model is not a good fit for the actual data.	

AR2, MA2, ARMA (1,1), ARMA(1,2) and ARMA(2,2) did not have all significant coefficients and were therefore not considered to be good models of the deseasonalized residuals. ARMA (2,1) had all coefficients as significant but did not reduce the sum of squares effectively, thus not fitting the data set well and was not further reviewed. AR1 and MA1 models had all coefficients as significant and were reviewed further. A review of the ACF and PACF plots for each model indicated that these two models reduced the lag coefficients below the critical value except between lags 11 and 13, where the models did not reduce the lag coefficient below critical. With the plots being similar between the models, the simplest model was selected. In this case, the AR1 model is the simplest model to understand while still addressing the significant lag 1 through 10 autocorrelations.

The assumptions of ARMA, in which the residuals are normally distributed, independent and with constant variance, are met as shown in Figure 4-52 below.

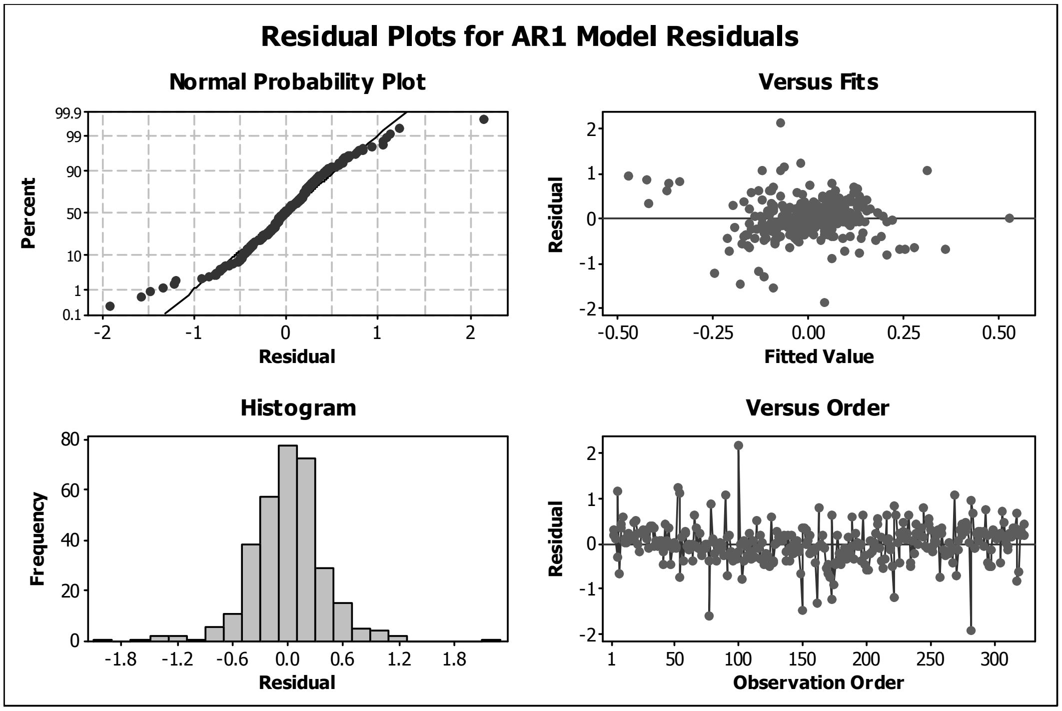


Figure 4-52: AR1 Model Residual Tests for Ugjoktok River

Simulation was used to verify the data set could indeed generate a data set of the same sample size with similar statistics. To verify the AR1 model, the model was required to reproduce the statistics of the actual data, which in this case are the residuals after the data set was deseasonalized. A Monte Carlo simulation of an AR1 process was completed whereby five hundred replications were used to determine if, on average, the AR1 model could reproduce the historical statistics. Below, in Table 4-19, is a comparison of the statistics for the actual deseasonalized data versus the AR1 simulated values.

Table 4-19: Comparison of AR1 Simulated Statistics to Statistics for Log-transformed Ugjoktok River Deasonalized Residuals

	N	Mean	Std dev	Min Flow	Max Flow	Skew	r1
Actual	324	0.00	0.44	-1.87	2.08	-0.31	0.25
Simulated	324	0.001	0.44	-1.27	1.27	-0.002	0.25
LC		-0.06	0.40	-1.60	0.94	-0.28	0.14
UC		0.06	0.47	-0.94	1.59	0.28	0.36

In order to state that the AR1 model for the deseasonalized flow data is able to reproduce the mean, standard deviation and lag one correlation on average to within the 95% confidence limits. As shown in Table 4-19 above, the mean, standard deviation and lag one correlation (r1) were within the 95% confidence interval. The other actual statistics should also be within the 95% confidence interval of the simulated statistics, however, for this model neither the minimum flow, maximum flow nor the skew were within the limits. Since these values were not explicitly modeled in the simulation, they would be difficult to preserve.

To graphically compare the actual data and the simulated data, time series plots for both were completed and are shown in Figure 4-53 and Figure 4-54 below. As shown, the time series plot for the simulated data looks similar to the time series plot for the actual data set, and as such, the AR1 model appears to be a good model representation of the deseasonalized data of the Ugjoktok River.

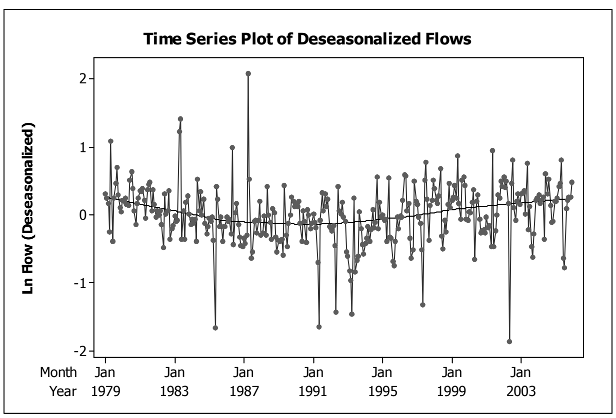


Figure 4-53: Time Series Plot of Deseasonalized Actual Data for Ugjoktok River

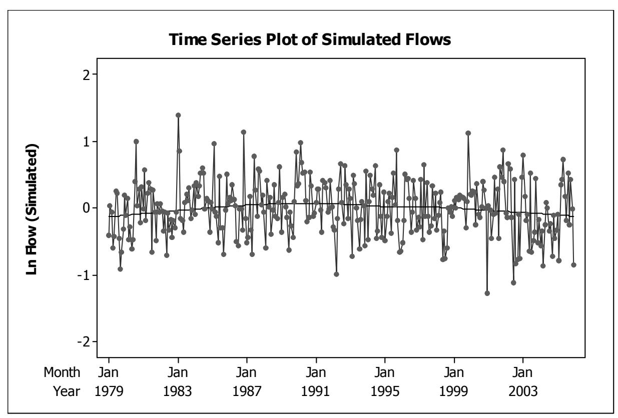


Figure 4-54: Time Series Plot of One AR1 Simulated Data Set for Ugjoktok River

4.2.2.3 Flow Simulation

Since the Ugjoktok River has potential as a hydroelectric development site, multiple simulations of flows can be used to estimate its energy potential. Simulation can be used to construct a set of flow duration curves for the river that can be used for capacity planning as well as reliability analysis. If a reservoir is planned for the site, the simulations can provide multiple inflow sequences to a reservoir model.

The following curves for the Ugjoktok River were developed using the Fourier 5 model results in conjunction with nine AR1 model simulations over 27 years, from 1979-2005 inclusive. These flow duration curves are shown in Figure 4-55 and together illustrate the range of flow duration curves for the Ugjoktok River. For comparison, the flow duration curve based on the actual measured flows, from 1979-2005, is shown in Figure 4-56 and shows similarities to the flow duration curves developed using multiple simulations.

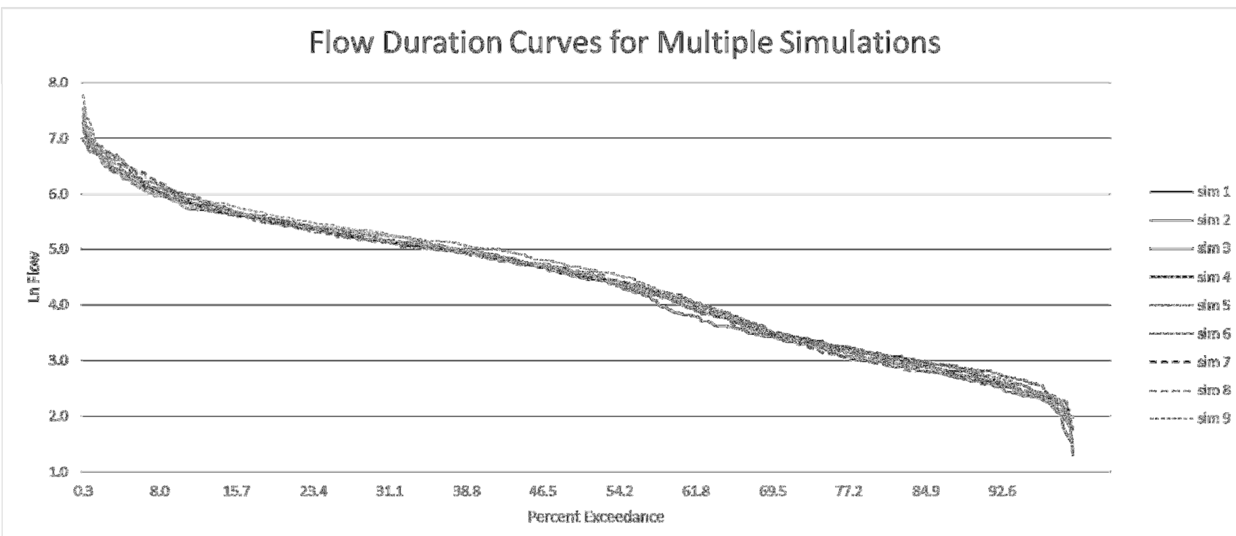


Figure 4-55: Simulated Flow Duration Curves for Ugjoktok River

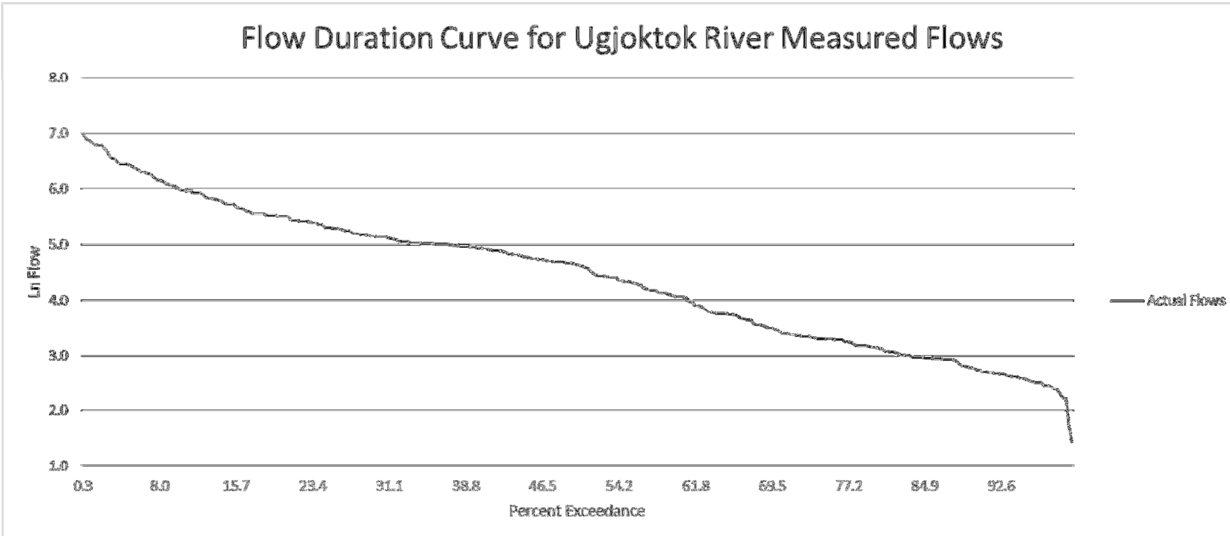


Figure 4-56: Flow Duration Curve for Ln Actual Ugjoktok River Flows

4.2.2.4 Forecasting

The Fourier model was used to forecast five years of flow values, from 2006 to 2010. These correspond to the years that were removed from the original data set prior to model development. Similar to the Alexis River forecasting, a period of five years was selected to forecast.

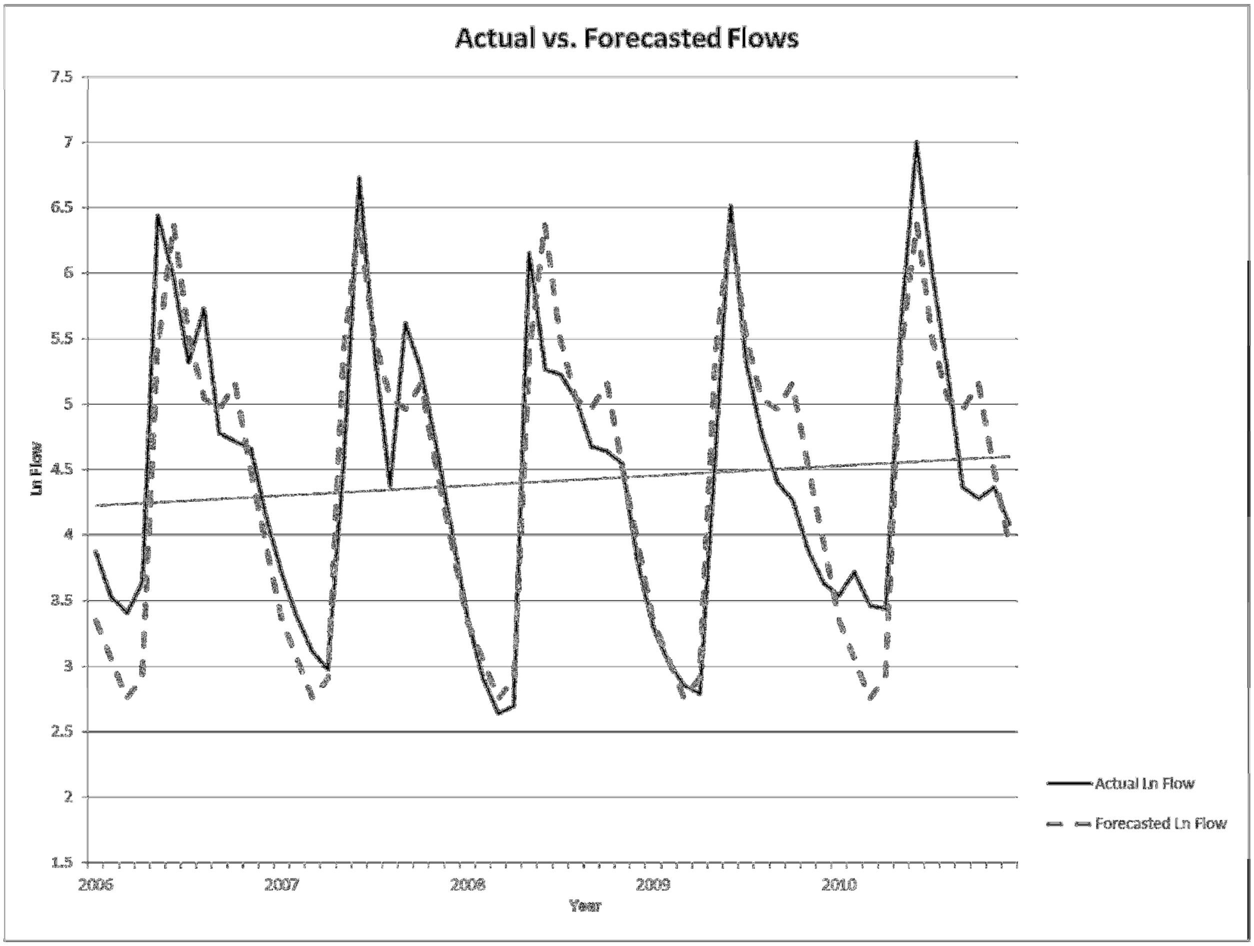


Figure 4-57: Actual vs Forecast Flows for Ugjoktok River (2006-2010)

As shown in Figure 4-57, the Box Jenkins model appears to do a reasonably good job of predicting the first three years (2006 to 2008) but does not fit the actual data well from July 2009 to March 2010 as well as Sept to Dec 2010. As was done for the Alexis River, three methods were used to validate the predictions: Nash–Sutcliffe efficiency, Mean Squared Deviation and Median Absolute Percentage Error. The actual data was segregated to match the Alexis River groupings of 2006-2008 and 2009-2010. Table 4-20 shows the error calculation results.

Table 4-20: Validation Results for ARMA with Spectral Analysis Forecasts of Ugjoktok River

Years in Review Set	NSE coefficient	MSD	MAPE
Full Set (1979–2010)	0.86	19.68	5.83%
2006–2010	0.81	22.58	6.34%
2006–2008	0.81	22.11	6.34%
2009–2010	0.81	23.28	6.93%

As shown in the table, the predicted flows between 2006 and 2010 show a value of 0.81 which is less efficient than the full data set at 0.87. As noted above, a graphical review of Figure 4-57 indicates that actual flows between the summer to winter of 2009 did not appear to display the same pattern as historical flows had shown; primarily there were higher actual flows during the winter period and lower flows during the summer than observed historically. As noted in the table above, the 2006–2008 predicted flows are a marginally better match to the actual data than are the 2006-2010 or 2009-2010. Based on these results, the selected model, generated using ARMA with the spectral analysis method, has been used to predict short term with reasonable accuracy.

4.2.2.5 Summary

Analysis of the monthly flows from the Ugjoktok River shows no trend but did show seasonality in the data set with three periods; 12 months, 6 month and 4 months. As is typical for hydrologic data sets, the data required a log transformation to improve normality and as such, the models were developed on the transformed data set. A time series analysis approach was selected to develop a model of the Ugjoktok River monthly data. In order to develop a comprehensive model of the flow, a combination approach was selected; model the seasonality using frequency domain spectral analysis and model the remaining residuals using the time domain ARMA model. A Fourier 5 sine and cosine pair model was selected to address the periodicity as it best addressed the seasonality in the data. An AR1 model was selected to model the remaining deseasonalized data as it best addressed the correlation found in the residuals. The AR1 model was able to reproduce the time series plot and the actual statistics within accepted tolerances.

The model was used to forecast flows with reasonable accuracy based on NSE, MSD and MAPE error calculations as well as to construct a range of flow duration curves.

4.2.3 Romaine River

Preliminary analysis of the Romaine River streamflows indicated that the data were not normally distributed as evidenced by a normality test and box plot, however, the data were found to be normal after a log transformation. Box plots showed seasonality in the data and there was no significant trend detected in the fitted line test. Tests for autocorrelation revealed there is short term dependence in the flow data for the Romaine River. The data from 1960 to 2005 were used to develop the model, with the last five years of the actual data set, 2006 to 2010, being used to compare the model forecast values to actual values.

4.2.3.1 Seasonal Analysis

When developing the model, the full actual data set was not used. Instead a fitting data set was used, from 1960 to 2005, to develop the model with the last five years of the actual data set removed. The last five years of actual data was used to verify the model.

This time series model included modeling seasonality, removing that seasonality and modeling the remaining residuals. In order to model the seasonality, the periods associated with periodic data sets were identified. Since the monthly Romaine River data were shown to have periodicity, the identification of the periods were accomplished using spectral analysis. A scatterplot of the spectral density function versus period is shown below in Figure 4-34. Similar to the spectral analysis completed for the Alexis and Ugjoktok rivers, the plot for the Romaine River suggests there are three periods in the data set; one at 12 months, one at 6 months and one at 4 months. Figure 4-58 is a scatterplot of the spectral density function versus frequency. This plot indicates there are three frequencies; one at 0.08, one at 0.17 and one a 0.25. Since the period is the inverse of frequency, the noted frequencies correspond to periods of 12 months, 6 months and 4 months as identified in Figure 4-59.

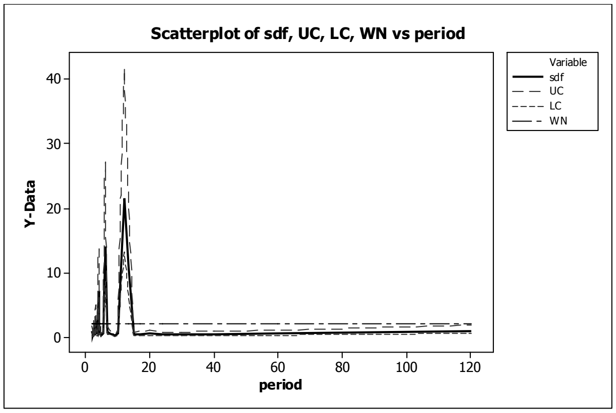


Figure 4-58: Scatterplot of Spectral Density Function versus Period for Romaine River

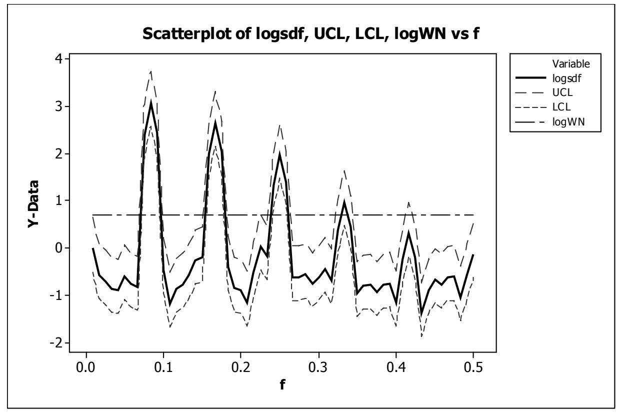


Figure 4-59: Scatterplot of Spectral Density Function versus Frequency for Romaine River

With seasonality and the periods confirmed by spectral analysis, the seasonal component of the data set was modeled with spectral analysis using the Fourier method. Analysis proved that the best fitting equation included 5 sine and cosine pairs, where all but one of the pairs are significant, the residual error is low and adjusted R^2 value is one of the highest of the models at 83.6%. As a result, the Fourier 5 pair model was selected. The results of the regression analysis are shown below in Table 4-21 and Table 4-22. The predictors listed in the table form part of the regression equation and in addition to a constant value, include sine and cosine pairs, indicating the ultimate number of pairs in the regression equation.

Table 4-21: Regression Results of Fourier 5 Sine and Cosine Pair Model for Romaine River

Predictor	Coefficient	SE Coefficient	T	P	Result
Constant	5.31543	0.01474	360.71	0.000	Significant
Sin(2pt)	-0.57271	0.02084	-27.48	0.000	Significant
Cos(2pt)	-0.64912	0.02084	-31.15	0.000	Significant
Sin(4pt)	-0.32567	0.02086	-15.61	0.000	Significant
Cos(4pt)	0.48450	0.02082	23.27	0.000	Significant
Sin(6pt)	0.16012	0.02084	7.68	0.000	Significant
Cos(6pt)	-0.23806	0.02084	-11.42	0.000	significant
Sin(8pt)	-0.17341	0.02086	-8.31	0.000	significant
Cos(8pt)	-0.00354	0.02082	-0.17	0.865	Not significant
Sin(10pt)	0.10147	0.02084	4.87	0.000	significant
Cos(10pt)	0.05961	0.02084	2.86	0.004	significant

Table 4-22: ANOVA Table of Fourier 5 Sine and Cosine Pair Model for Romaine River

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	Result
Regression	328.764	10	32.876	278.34	0.00	significant
Error	62.956	533	0.118			
Total	391.720	543				

The regression model equation is as shown below in Eqn. [26]. The assumptions of ANOVA, in which the residuals are normally distributed, independent and with constant variance, are met as shown in Figure 4-60 below.

Ln Flow = 5.32 – 0.573 sin(2pt) - 0.649 cos(2pt) - 0.326 sin(4pt) + 0.485 cos4pt + 0.160 sin(6pt) - 0.238 cos(6pt) - 0.173 sin(8pt) - 0.0035cos(8pt) + 0.101 sin(10pt) + 0.0596 cos(10pt) [28]

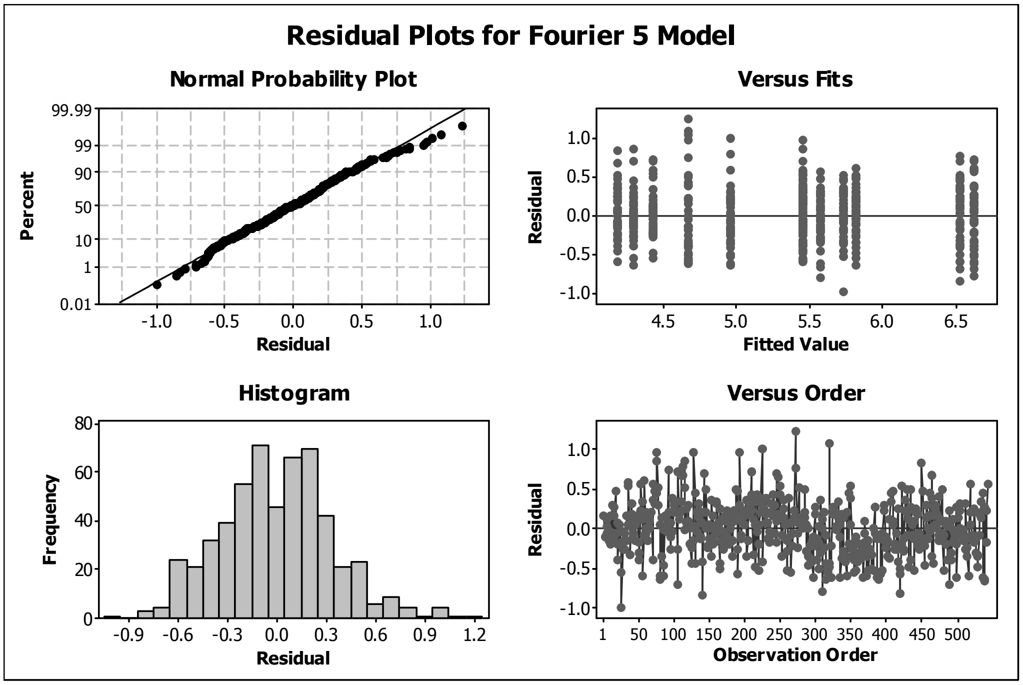


Figure 4-60: Fourier 5 Model Residual Tests for Romaine River

To verify the seasonal model, the fitted function values were compared to the actual data for the record from 1960 to 2005. As shown in Figure 4-61, the fitted function models the actual values fairly well.

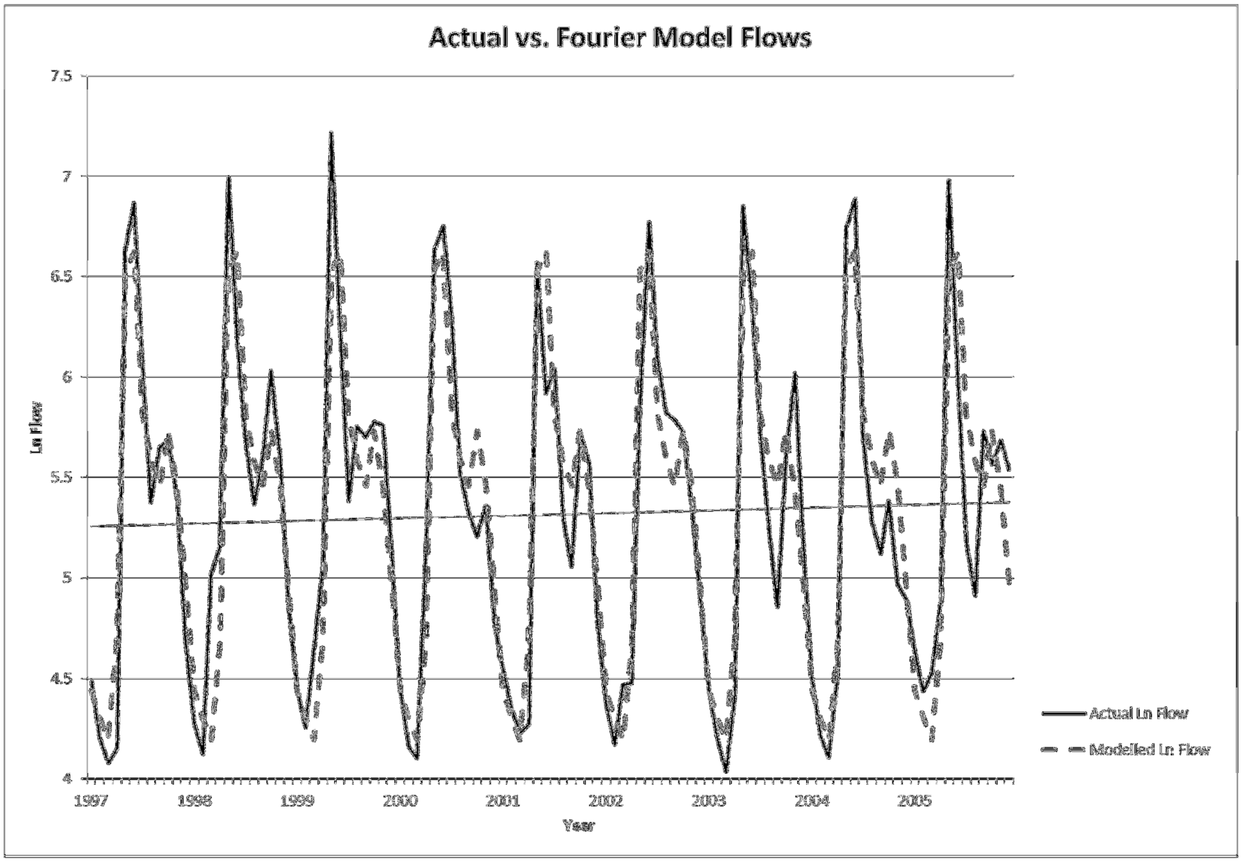


Figure 4-61: Actual Monthly Flows versus Fourier Model Values for Romaine River

The error between the fitted function values and the actual values was calculated using the NSE coefficient, MSD and MAPE. The error results are provided in Table 4-23.

Table 4-23: Calculated Error between Fourier Fitted Function and Actual Flow Values for Romaine River

Years in data set	NSE Coefficient	MSD	MAPE
1960–2005	0.84	11.57	4.04%

ACF and PACF plots were used to determine if there was any remaining seasonality in the residuals. In addition, the remaining lag correlation indicated the type of ARMA model that was required to address the autocorrelation. As shown in Figure 4-62 and Figure 4-63, the seasonality was removed from the Fourier

5 model residuals, however there remained a lag 1 correlation and as such, further ARMA modeling was required to address the strong autocorrelation in the residuals.

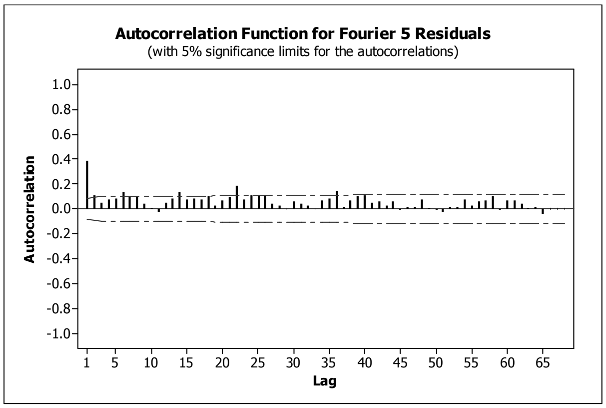


Figure 4-62: ACF Plot of Residuals for Romaine River

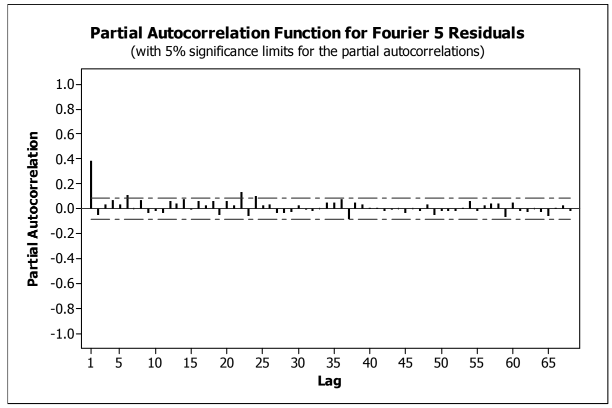
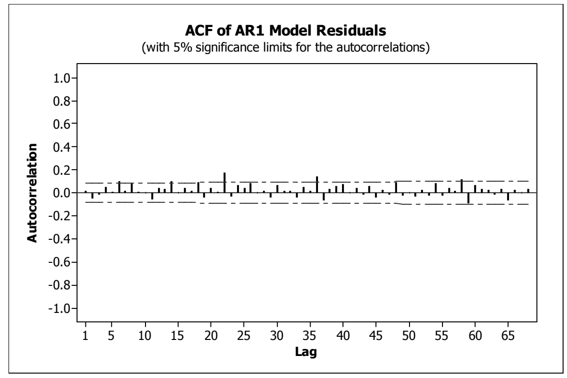


Figure 4-63: PACF Plot of Residuals for Romaine River

4.2.3.2 ARMA Analysis of Deseasonalized Flows

Since the seasonality was removed from the Romaine River monthly data set using Spectral Analysis and Fourier Modeling, the remaining deseasonalized data were modeled using ARMA. As previously shown in the ACF and PACF of the residuals in Figure 4-62 and Figure 4-63, there is a lag 1 correlation in the PACF plot which suggests an AR1 model might be most appropriate. For completeness, a number of different ARMA models were completed and the statistics of each reviewed to determine the best fitting ARMA model for the Romaine actual data set. Table 4-24 summarizes the results of the reviewed ARMA models complete with corresponding ACF and PACF plots of the model residuals.

Table 4-24: Results of Different ARMA Models for Romaine River

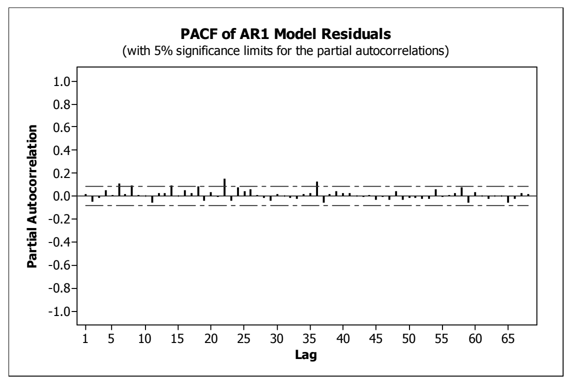
Model Type	Model Results	ACF and PACF PLots
AR1	Type / Coef / SE Coef / T / P AR 1 / 0.3877 / 0.0397 / 9.76 / 0.000 Constant / 0.00052 / 0.01348 / 0.04 / 0.969 Mean / 0.00085 / 0.02201	

Type	Coef	SE Coef	T	P
AR 1	0.3877	0.0397	9.76	0.000
Constant	0.00052	0.01348	0.04	0.969
Mean	0.00085	0.02201		

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	15.2	49.7	72.9	91.2
DF	10	22	34	46
P-Value	0.124	0.001	0.000	0.000

AR1 coefficient is significant however there are significant lag correlations remaining around lag 22 and 36. Only one of the Ljung-Box test results is not significant, indicating independence and randomness of the data. The remaining significant Ljung –Box tests indicate dependence. This model will be compared to other models with significant coefficients.



Model Type	Model Results	ACF and PACF PLots

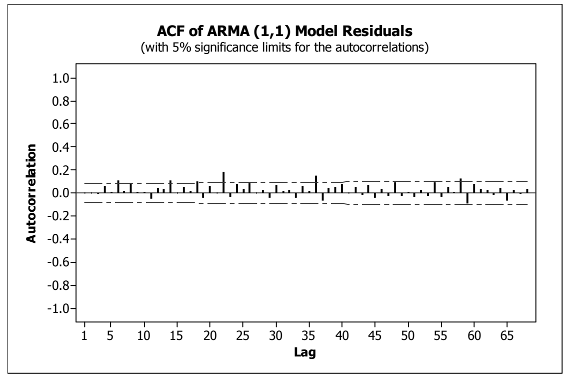
ARMA (1,1)

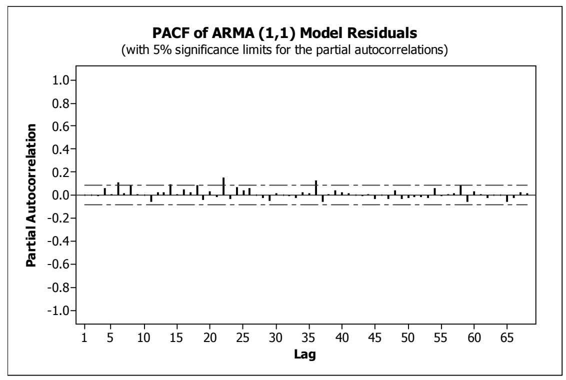
Type	Coef	SE Coef	T	P
AR 1	0.2723	0.1054	2.58	0.010
MA 1	-0.1360	0.1084	-1.25	0.210
Constant	0.00053	0.01530	0.03	0.972
Mean	0.00073	0.02103		

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	15.2	54.6	79.9	100.4
DF	9	21	33	45
P-Value	0.086	0.000	0.000	0.000

The AR1 coefficient is significant however, the MA1 coefficient is not significant therefore this model is not a good fit for the actual data.



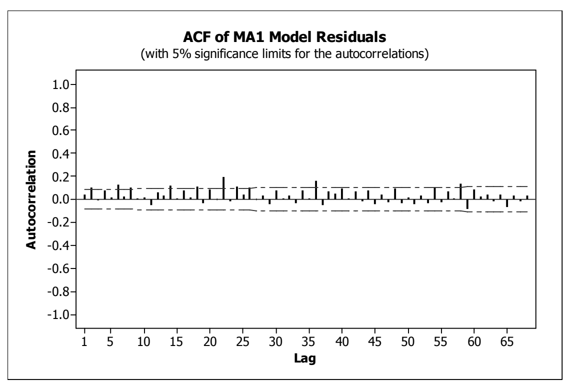


MA 1

Type	Coef	SE Coef	T	P
MA 1	-0.3676	0.0400	-9.20	0.000
Constant	0.00042	0.01851	0.02	0.982
Mean	0.00042	0.01851		

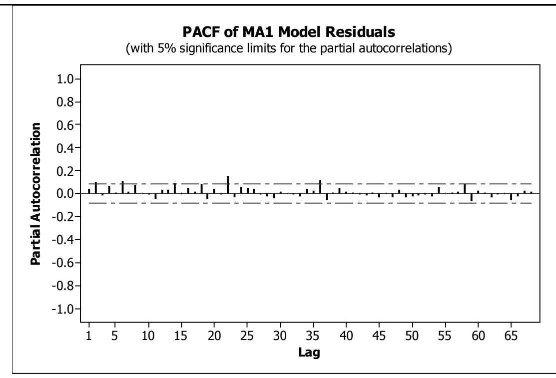
Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	27.5	78.7	109.2	134.0
DF	10	22	34	46



Model Type	Model Results	ACF and PACF PLots

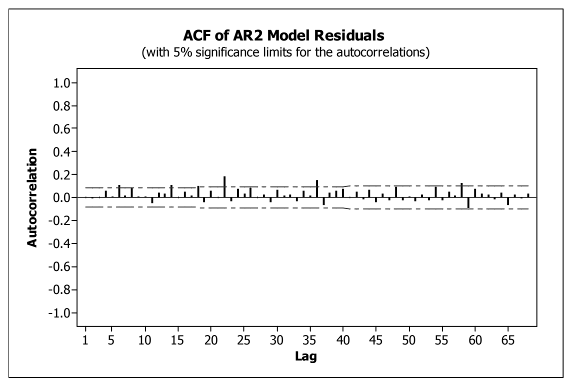
P-Value	0.002	0.000	0.000	0.000



MA1 coefficient is significant however there are significant lag correlations remaining around lags 6, 14, 18, 22 and 36. The Ljung –Box tests are significant suggesting that the data are not independent or random. This model will be compared to other models with significant coefficients.

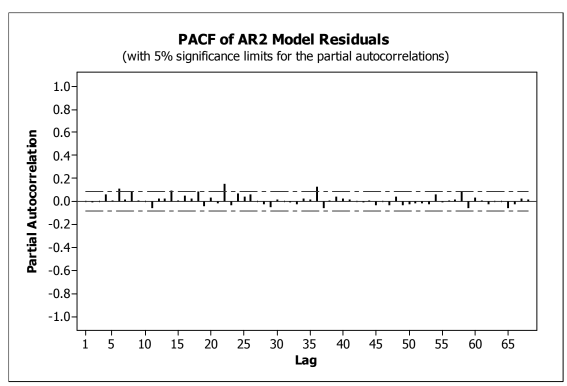
AR 2

Type	Coef	SE Coef	T	P
AR 1	0.4071	0.0430	9.46	0.000
AR 2	-0.0501	0.0430	-1.16	0.245
Constant	0.00047	0.01347	0.03	0.972
Mean	0.00073	0.02095		



Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	15.1	54.0	79.1	99.3
DF	9	21	33	45
P-Value	0.088	0.000	0.000	0.000



The AR1 coefficient is significant but the AR2 coefficient is not, therefore this model is not a good fit for the actual data.

Model Type	Model Results	ACF and PACF PLots

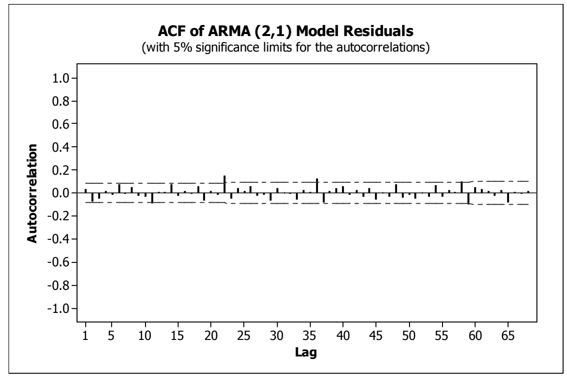
ARMA 2,1

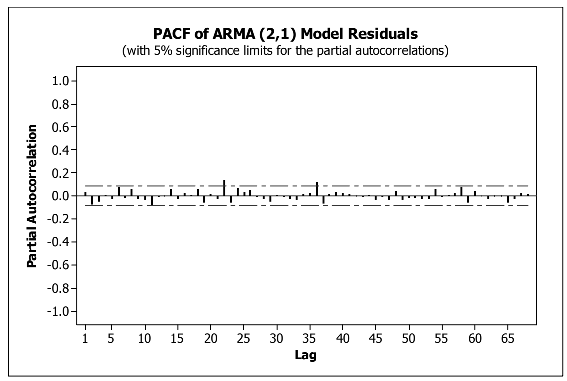
Type		Coef	SE Coef	T	P
AR	1	1.3212	0.0408	32.41	0.000
AR	2	-0.3310	0.0408	-8.12	0.000
MA	1	0.9621	0.0003	2984.06	0.000
Const		0.0000111	0.0006857	0.11	0.987
Mean		-0.00113	0.06971		

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	15.0	380.8	56.8	72.0
DF	8	20	32	44
P-Value	0.060	0.007	0.004	0.005

All coefficients are significant. The first Ljung-Box test is not significant indicating independence and randomness for lag 12. This model will be compared with other models with significant coefficients.





ARMA 1,2

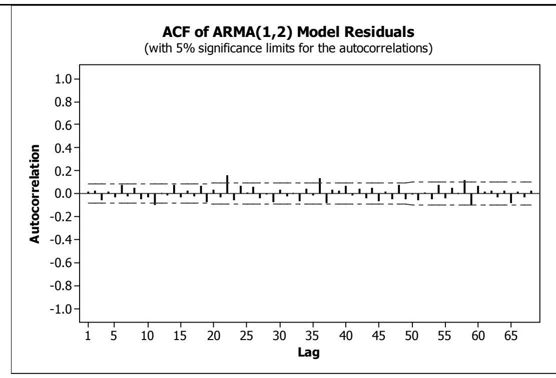
Type		Coef	SE Coef	T	P
AR	1	0.9815	0.0211	46.54	0.000
MA	1	0.6140	0.0510	12.04	0.000
MA	2	0.3163	0.0422	7.50	0.000
Constant		-0.000027	0.000934	-0.03	0.977
Mean		-0.00146	0.05034		

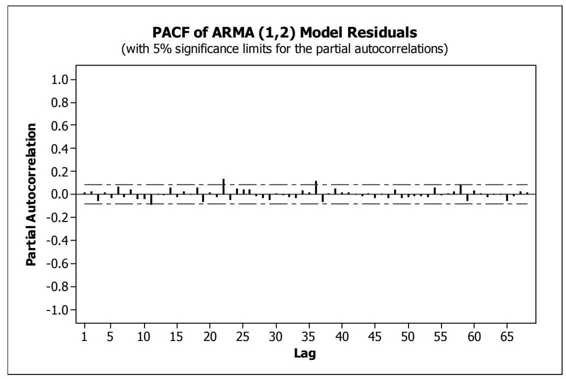
Model Type	Model Results	ACF and PACF PLots

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	15.8	45.6	66.8	85.5
DF	8	20	32	44
P-Value	0.046	0.001	0.000	0.000

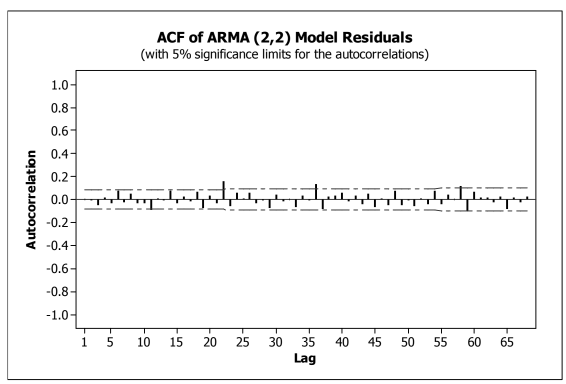
All ARMA 1,2 coefficients are significant however all Ljung Box statistics are significant indicating that the data are not independent or random. The model was also not able to reduce sum of squares efficiently. This model will be compared to other models with significant coefficients



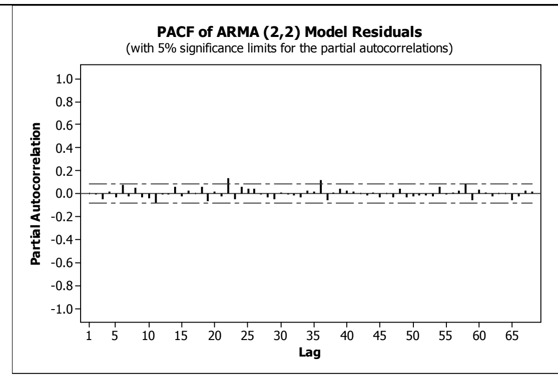


ARMA 2,2

Type	Coef	SE Coef	T	P
AR 1	1.1141	0.2062	5.40	0.000
AR 2	-0.1287	0.1388	-0.93	0.354
MA 1	0.7283	0.2057	3.54	0.000
MA 2	0.2159	0.0839	2.58	0.010
Const	-0.000024	0.00077	-0.03	0.975
Mean	-0.00161	0.05248		



Model Type	Model Results	ACF and PACF PLots



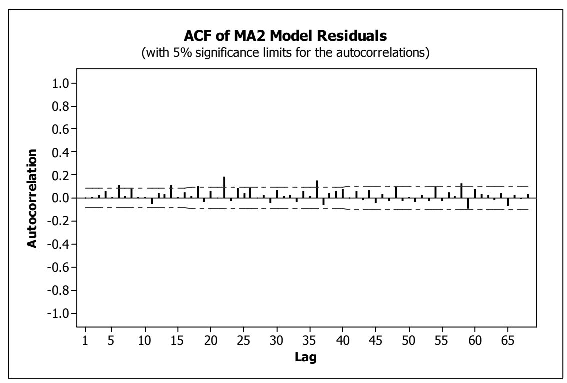
Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	13.0	41.3	61.5	79.3
DF	7	19	31	43
P-Value	0.073	0.002	0.001	0.001

The AR1, MA1 and MA2 coefficients are significant however the AR2 coefficient is not, therefore this model is not a good fit for the actual data.

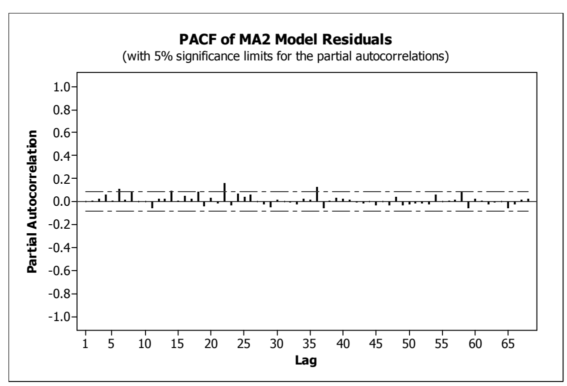
MA2

Type	Coef	SE Coef	T	P
MA 1	-0.4095	0.0429	-9.55	0.000
MA 2	-0.1018	0.0430	-2.37	0.018
Constant	0.00068	0.02036	0.03	0.973
Mean	0.00068	0.02036		



Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	15.5	55.1	80.4	100.4
DF	9	21	33	45
P-Value	0.078	0.000	0.000	0.000



Both the MA1 and MA2 coefficients are significant and one of the Ljung Box statistics are significant

Model Type	Model Results	ACF and PACF PLots
	indicating the data are independent and random at lag 12. There are significant lag correlations around lag 6, 14, 22 and 36. This model will be compared to other models with significant coefficients.	

AR2, ARMA 2,2 and ARMA 1,1 had some significant coefficients but not all were significant and therefore theses models were not considered to be a good fit of the deseasonalized data. ARMA 1,2 and MA1 had all coefficients as significant, however all of the four Ljung-Box Chi Square statistics were significant, indicating the data were not independent or random. The remaining three best fitting models, AR1, MA1 and ARMA 2,1 had all significant as well as the lag 12 Ljung-Box Chi Square statistic not significant, suggesting independence. A review of the ACF and PACF plots for each model indicated that the three models were effective in reducing most of the lag coefficients below the critical value except lags 22 and 36, where all three models did not reduce the lag coefficient below critical. The MA2 model also had significant correlations remaining at lags 6, 14, 22 and 36. The AR1 model and the ARMA 2,1 models only had significant lags remaining at 22 and 36 suggesting that the MA2 model was not as good a fit as the AR1 or ARMA 2,1 models. With the AR1 and the ARMA 2,1 models being relatively comparable in fitting the data set, the AR1 model was selected as the preferred model since it was a simpler model.

The AR1 model results, as shown above in Table 4-24, indicate that the AR1 parameter was significant. The ACF and PACF plots of the AR1 model residuals indicate that correlations have been addressed and thus the AR1 model is good at modeling the residuals of the Romaine River deseasonalized monthly data. The assumptions of ARMA in which the residuals are normally distributed, independent and with constant variance, are met as shown in Figure 4-64 below.

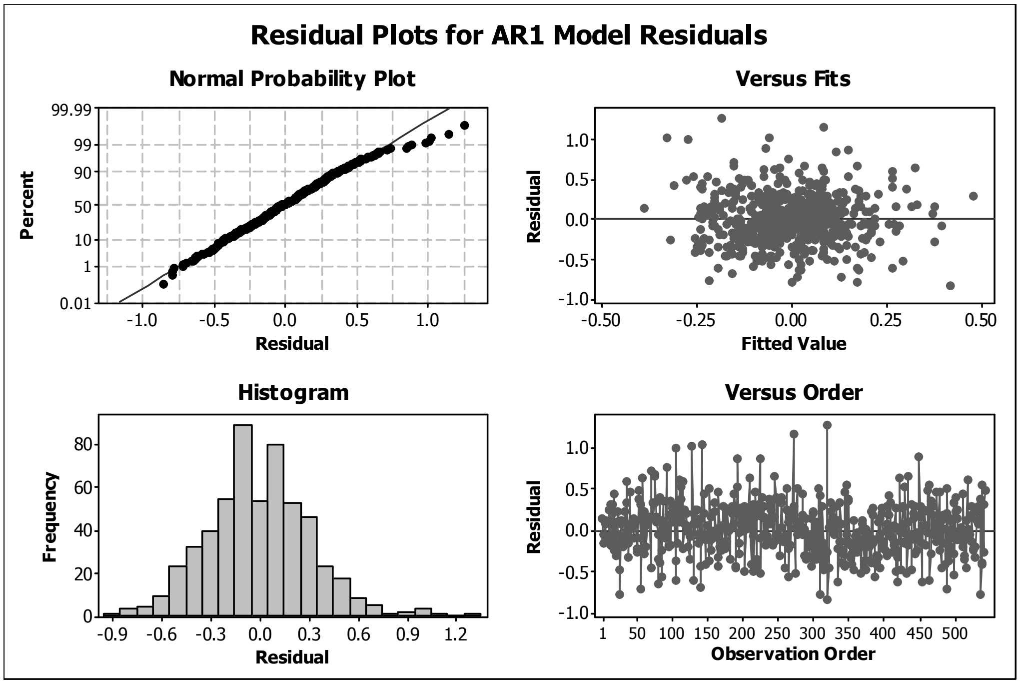


Figure 4-64: AR1 Model Residual Tests for the Romaine River

Simulation was used to verify that the data set could indeed generate a data set of the same sample size with similar statistics. To verify the AR1 model, the model was required to reproduce the statistics of the actual data, which in this case are the residuals after the data set was deseasonalized. A Monte Carlo simulation of an AR1 process was completed whereby five hundred replications were used to determine if, on average, the AR1 model could reproduce the historical statistics. Below, in Table 4-25, is a comparison of the statistics for the actual deseasonalized data versus the AR1 simulated values.

Table 4-25: Comparison of AR1 Simulated Statistics to Statistics for Log-transformed Romaine River Deasonalized Residuals

	N	Mean	Std dev	Min Flow	Max Flow	Skew	r1
Actual	544	0.00	0.34	-1.00	1.23	0.20	0.3855
Simulated	544	-0.0005	0.34	-1.04	1.04	-0.01	0.3782
LCL		-0.05	0.31	-1.3	0.79	-0.22	0.2976
UCL		0.04	0.36	-0.78	1.28	0.19	0.4589

An acceptable AR1 model should reproduce the mean, standard deviation and lag one correlation on average to within the 95% confidence limits. As shown in Table 4-25, the mean, standard deviation and lag one correlation (r1) were within the 95% confidence interval. The other actual statistics should also be within the 95% confidence interval of the simulated statistics, however, for this model only the minimum and maximum flows were within the limits while the skew was not. Since these values were not explicitly modeled in the simulation, they would be difficult to preserve.

To graphically compare the actual data and the simulated data, time series plots for both were completed and are in Figure 4-65 and Figure 4-66 below. As shown, the time series plot for the simulated data looks similar to the time series plot for the actual data set, and the AR1 model therefore appears to be a good model representation of the deseasonalized data of the Romaine River.

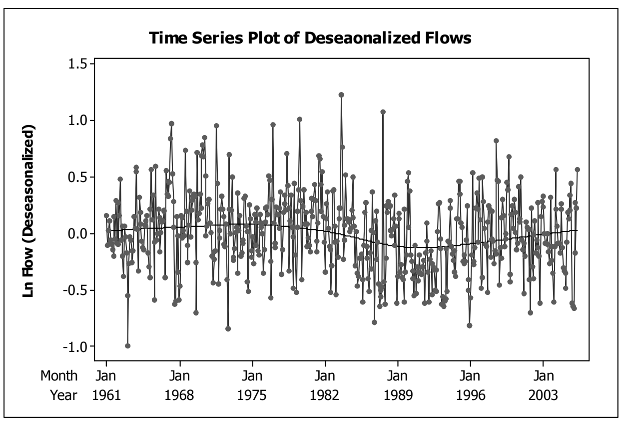


Figure 4-65: Time Series Plot of Deseasonalized Actual Data for Romaine River

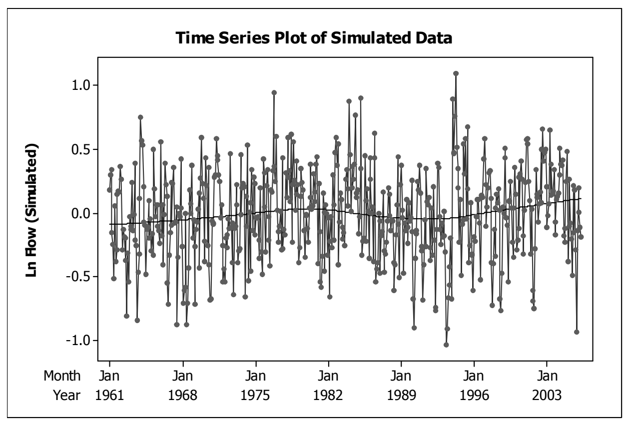


Figure 4-66: Time Series Plot of One AR1 Simulated Data Set for Romaine River

4.2.3.3 Flow Simulations

The Romaine River is the site of a number of hydroelectric developments currently under construction where multiple simulations of flows can be used to estimate its energy potential. Simulation can be used to construct a set of flow duration curves for the river that can be used for capacity planning as well as reliability analysis.

The following curves for the Romaine River were developed using the Fourier 5 model results in conjunction with nine AR1 model simulations for 45 years, from 1960 to 2005. These flow duration curves are shown in Figure 4-67 and together illustrate a range of flow duration curves for the Romaine River. For comparison, the flow duration curve based on the actual measured flows, from 1960 to 2005, is shown in Figure 4-68 and shows similarities to the flow duration curves developed using multiple simulations.

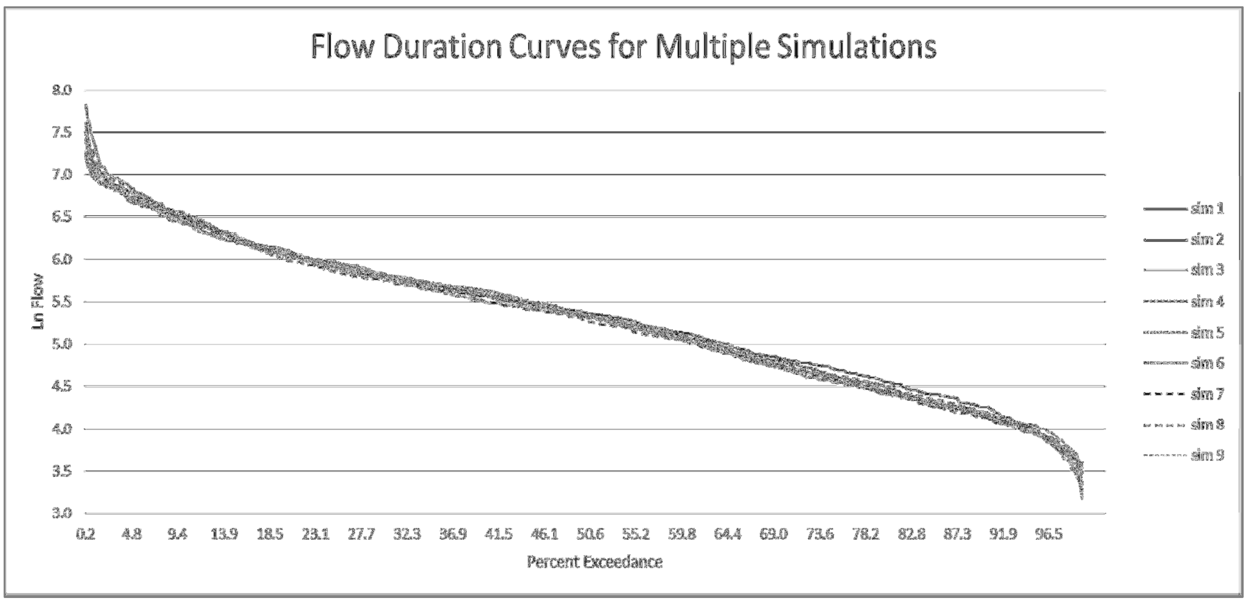


Figure 4-67: Simulated Flow Duration Curves for Romaine River

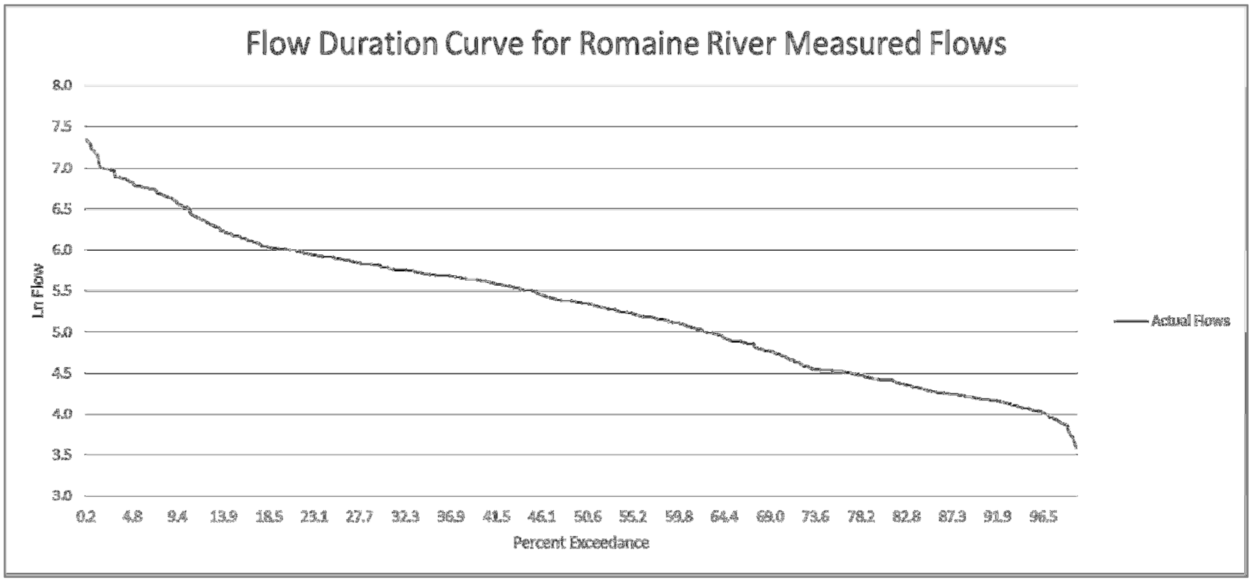


Figure 4-68: Flow Duration Curve for Ln Actual Romaine River Flows

4.2.3.4 Forecasting

The Fourier model was used to forecast five years of flow values, from 2006 to 2010. These correspond to the years that were removed from the original data set prior to model development. Similar to the Alexis and Ugjoktok River forecasting, a period of five years was selected to forecast.

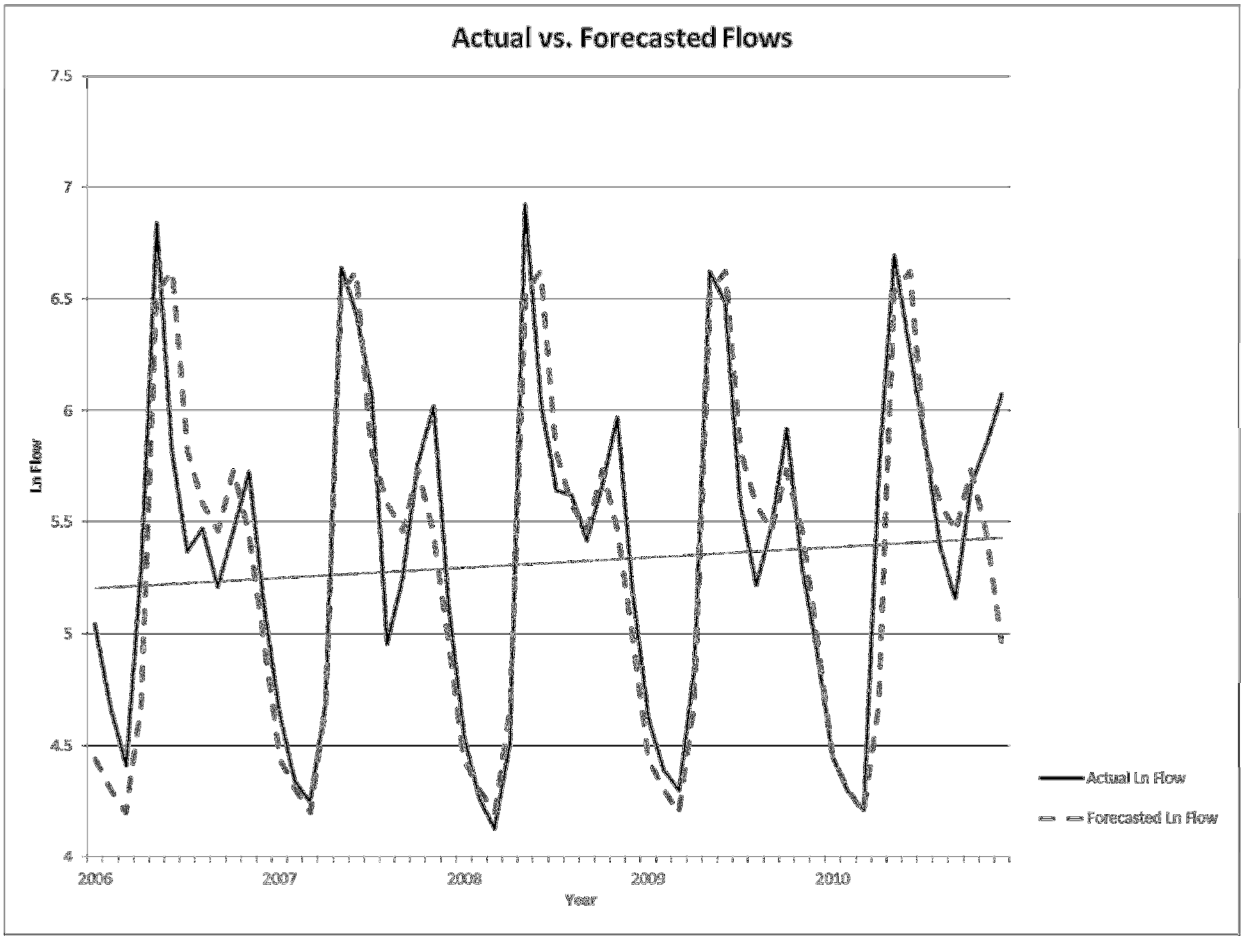


Figure 4-69: Actual vs Forecast Flows for Romaine (2006-2010)

The model appears to do a reasonably good job of predicting all five years with the exception of the last 3 months of 2010. To verify the prediction accuracy of the model three methods were used to validate the predictions: Nash–Sutcliffe efficiency, Mean Squared Deviation and Median Absolute Percentage Error. Years were grouped to match the Alexis and Ugjoktok River groupings for comparison purposes. Table 4-26 shows the error calculation results.

Table 4-26: Validation Results for ARMA with Spectral Analysis Forecasts of Romaine River

Years in Review Set	NSE coefficient	MSD	MAPE
Full Set (1960–2010)	0.84	11.61	4.01%
2006–2010	0.78	11.94	3.36%
2006–2008	0.79	11.15	4.13%
2009–2010	0.77	13.13	2.70%

As shown in the table, the predicted flows between 2006 and 2010 show a NSE coefficient value of 0.78 which is marginally less efficient than the full data set at 0.84. Figure 4-69 suggests that forecast values closely model the actual flows between 2006 and 2010. The NSE coefficient values as well as the MSD values are consistent and show the 2006–2008 predicted flows better match the actual data than do the values for 2006-2010 or 2009-2010. These findings are not supported using MAPE, with the MAPE results suggesting the forecasts were not as closely modelled for the 2006-2008 period as compared to the 2009-2010 period. It should be noted, however, that 2.70% and 4.13% error for the 2009-2010 and 2006-2008 records respectively, is a small margin of error and indicates that the deseasonalized ARMA model is an accurate predictor of future short term flows for the Romaine River.

4.2.3.5 Summary

Analysis of the monthly flows from the Romaine River shows no significant trend but does show seasonality in the data set with three periods: 12 months, 6 month and 4 months. These periods are the same periods that were discovered for the Alexis and Ugjoktok Rivers. As is typical for hydrologic data sets, the data required a Log-transformation to improve normality and as such, the models were developed on the transformed data set. A time series analysis approach was selected to develop a model of the Romaine River monthly data. In order to develop a comprehensive model of the flow, a combination approach was selected: model the seasonality using frequency domain spectral analysis and model the remaining residuals using the time domain ARMA model. Like the spectral analysis for Alexis and Ugjoktok Rivers, a Fourier 5 sine and cosine pair model was selected to address the periodicity as it best addressed the seasonality in the data set with an 83.6% adjusted R^2 for the regression. An AR1 model was selected to model the remaining deseasonalized residuals as it best addressed the correlation found in the residuals and was the simplest model. The AR1 model was able to reproduce the time series plot and the actual statistics within accepted tolerances. The model was used to forecast flows with reasonable accuracy based on NSE, MSD and MAPE error calculations as well as to construct a range of flow duration curves.

4.3 State Space Time Series Analysis

As described in Section 3.2, the STAMP software was used to develop state-space time-series models of the Alexis, Ugjoktok and Romaine rivers using observation and state equations as outlined in Harvey (1989). The approach in this study consisted of completing multiple trials of models to determine which combination of state space components produced the best fitting model. Once the best fitting model was selected for each river, short term forecasts were completed using the preferred model.

Similar to the Box Jenkin method, the last five years of the original data set was removed from the record for each river prior to development of the models, with those removed years being available for comparison with the forecasts from each of the selected models. The calculated error between the actual flows and the fitted and predicted flows determined each model's ability to fit and predict the actual data set.

This section details the diagnostic results of the multiple models completed for each river, the selection of the best fitting SSTS model for the rivers and the verification of the fitted and forecasted values.

4.3.1 Alexis River

As previously noted in Section 4.1.1, Alexis River flow data approximately match a lognormal distribution and as such, the Log-transformed data set was used in the SSTS analysis. The data set used for SSTS model development did not include the last five years of the flow record; the data range used was from 1979 to 2005. The flows from 2006 to 2010 were used to compare against the forecasted values and calculate accuracy of SSTS model prediction.

4.3.1.1 Model

A number of models were developed for Alexis River. For each model, the diagnostics, graphs and AIC value were reviewed. The best fitting model would have diagnostic tests within critical values and the model with the lowest AIC value would generally be the best solution. In addition, the variance disturbance values in the model results provided some direction as to whether the component disturbance was deterministic or stochastic.

As an example, one of the models produced for Alexis River included a deterministic level and an irregular component. The component graphics for this model are found in Figure 4-70.

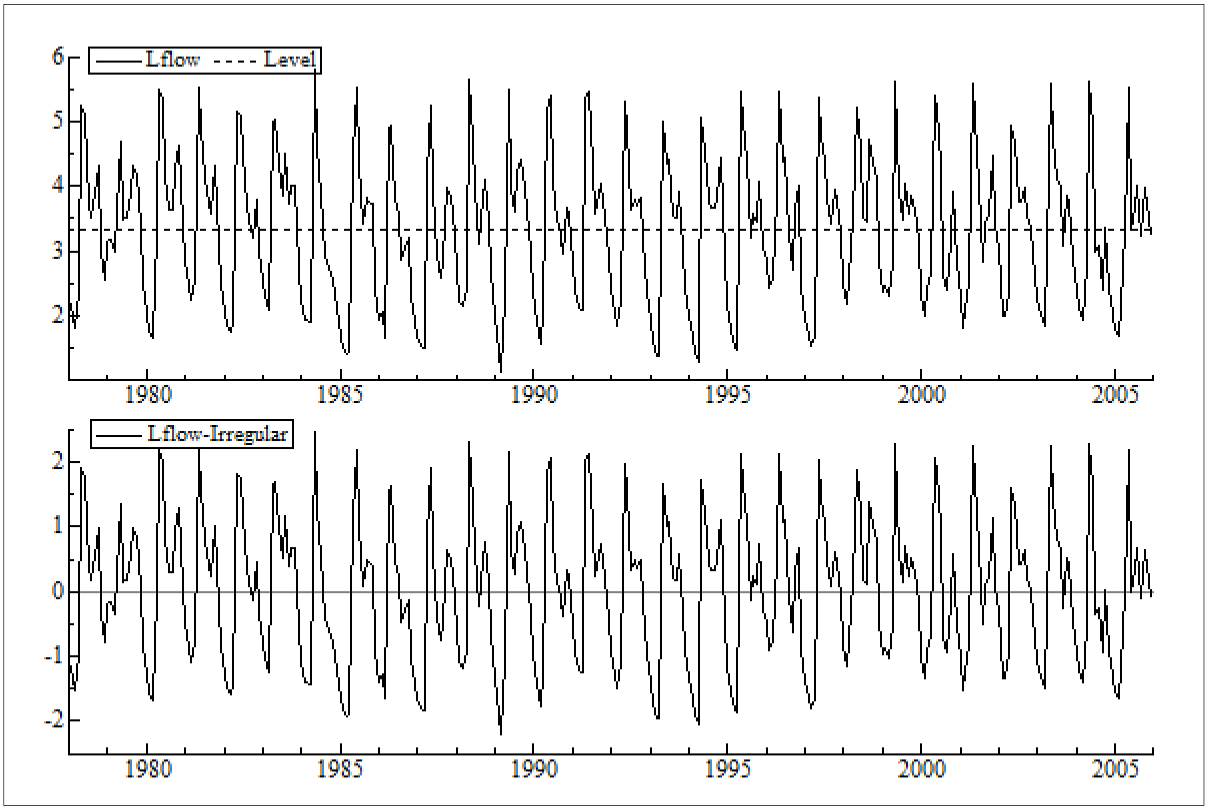


Figure 4-70: Level and Irregular Component Graphics for Alexis River SSTS Deterministic Level Model

As shown in the above figure, the irregular component appears to display a pattern, suggesting this model could be improved to account for the pattern. In addition, a review of the ACF plot, Figure 4-71, of the model residuals showed a sine wave pattern, suggesting seasonality in the model residuals.

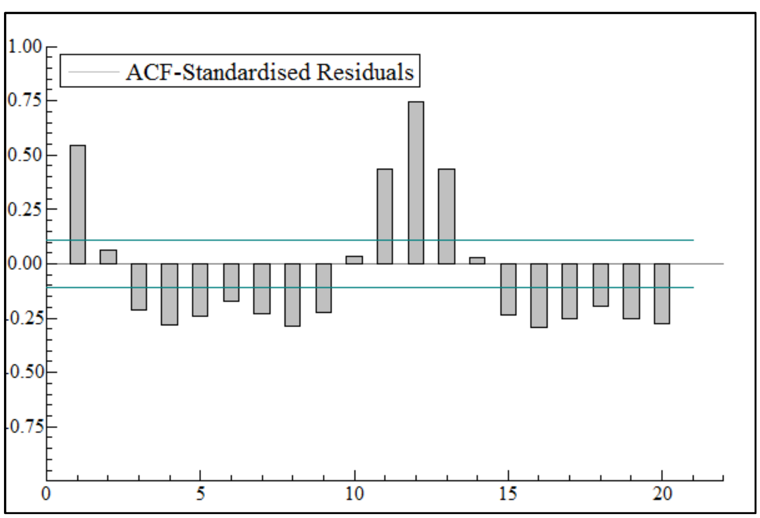


Figure 4-71: ACF Plot of SSTS Deterministic Level Model Residuals for Alexis River

Another one of the models produced included a stochastic level, stochastic slope, stochastic seasonal and irregular component. The component graphics for this model are found in Figure 4-72 .

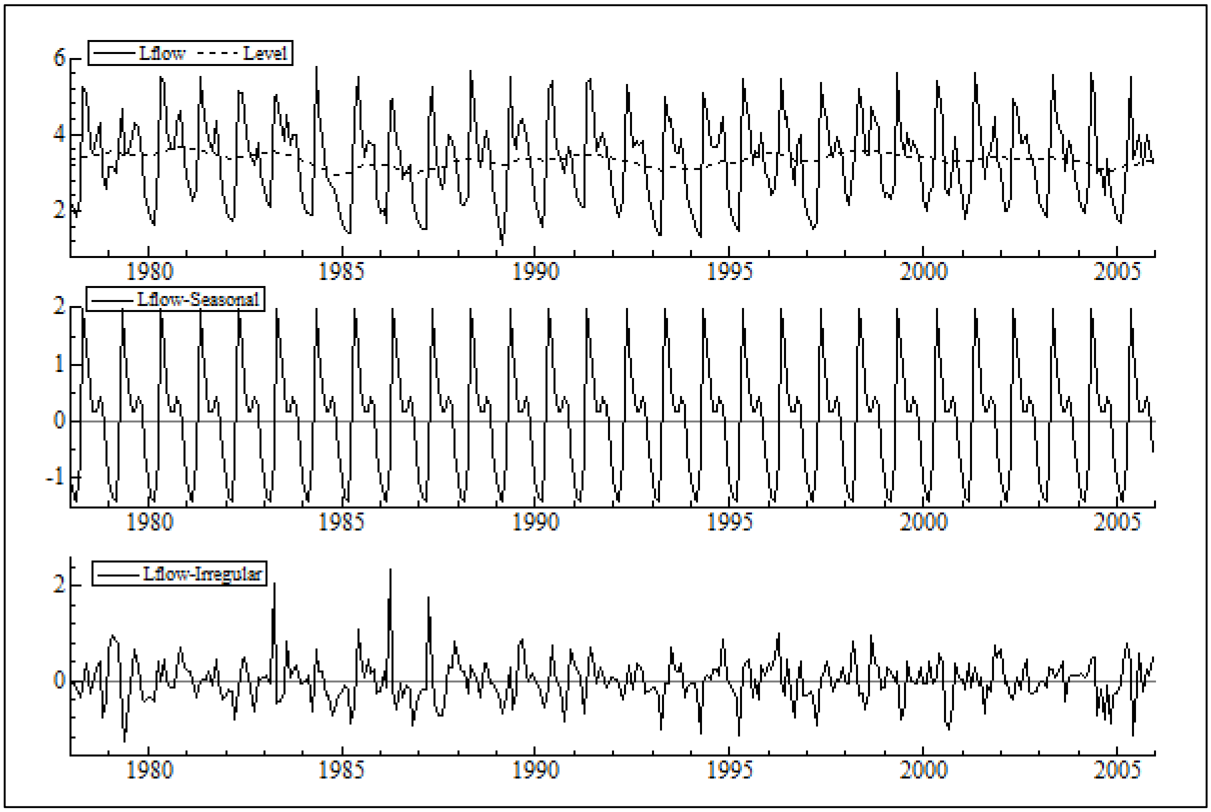


Figure 4-72: Level, Seasonal and Irregular Component Graphics for Alexis River SSTS Model

The undulating dashed line in the top graph of Figure 4-72 indicated a stochastic level baseline. This was confirmed in the model results whereby the variance of disturbance of the level term was not zero. The seasonal (middle) graph appeared to have a regular pattern and there did not appear to be a pattern in the irregular (bottom) graph, suggesting only white noise remained in the irregular component of this model.

A number of combinations of components were modelled and the diagnostics as well as graphics were examined. Table 4-27 outlines the component combinations for the developed models.

Table 4-27: SSTS Model Combinations for Alexis River

Model #	level		slope		seasonal		irregular
	fixed	**stochastic**	**fixed**	**stochastic**	**fixed**	**stochastic**	
1	X						X
2		X					X
3	X		X				X
4		X		X			X
5	X				X		X
6		X				X	X
7		X	X		X		X
8		X		X		X	X
9		X			X		X

For each of the above noted models, the diagnostics were separately reviewed. The critical values were calculated in a spreadsheet and compared against the model results. Table 4-28 gives a summary of the critical values calculated for each diagnostic test along with the AIC calculation for each developed model.

Table 4-28: Diagnostic Tests for Alexis River SSTS Models

	Model #	1	2	3	4	5	6	7	8	9
r(1)	Result value	0.544	0.029	0.546	0.030	0.375	0.304	0.275	0.275	0.304
	Critical value	0.109	0.109	0.109	0.109	0.109	0.109	0.109	0.109	0.109
	assumption	Not met	Met	Not met	met	Not met	Not met	Not met	Not met	Not met
r(q)	Result value	0.733	0.634	0.719	0.637	-0.010	-0.038	-0.037	-0.037	-0.038
	Critical value	0.109	0.109	0.109	0.109	0.109	0.109	0.109	0.109	0.109
	assumption	Not met	Not met	Not met	Not met	met	met	met	met	Met
Q(q,q-p)	Result value	981.00	459.60	952.07	459.12	66.695	59.467	53.523	53.523	59.467
	Critical value	35.172	35.172	33.924	33.924	33.924	33.924	32.671	32.671	33.924
	assumption	Not met	Not met	Not met	Not met	Not met	Not met	Not met	Not met	Not met
H(h)	Result value	1.199	1.007	1.206	1.016	1.888	1.805	1.814	1.814	1.805
	Critical value	1.984	1.984	1.984	1.984	1.984	1.984	1.984	1.984	1.984
	assumption	Met	Met	Met	Met	Met	Met	Met	Met	Met
N	Result value	14.515	276.98	24.430	260.05	14.367	21.954	25.791	25.791	21.954
	Critical value	5.991	5.991	5.991	5.991	5.991	5.991	5.991	5.991	5.991
	assumption	Not met	Not met	Not met	Not met	Not met	Not met	Not met	Not met	Not met
AIC		0.230	0.140	0.236	0.146	-1.234	-1.246	-1.239	-1.239	-1.246
convergence		yes	Very strong	yes	Very strong	yes	Very strong	Very strong	Very strong	Very strong
Variance of disturbance	Level		1.1403		1.1437		0.0029	0.0063	0.0063	0.0029
	Slope				0				0	
	Seasonal						0		0	

where r(1) is the serial correlation and r(q) is serial correlation lag at step q, Q(q, q–p) is the Box-Ljung Q-statistic, H is the test for heteroscedasticity, N is a test for normality (Koopman *et al.* 2009), and AIC is the Akaike Information Criterion (Akaike 1974).

A review of Table 4-28 shows that models 6 and 9 result in the lowest AIC value. As identified in Table 4-27, model 6 included stochastic level, stochastic seasonal and irregular components while model 9 included stochastic level, deterministic seasonal and irregular components. An examination of the variance of disturbance results indicated that for model 6, the variance of disturbance for the seasonal component was zero. This discovery suggests that the seasonal component, though selected in this model as stochastic, was in fact deterministic, meaning model 6 and model 9 were in fact the same model with a deterministic seasonal component. Based on AIC and variance review, model 9 was potentially the best fitting model.

An evaluation of the component graphics, as shown in Figure 4-73, indicated that the level equation is stochastic and evidenced by the undulating dashed line in the top graph. The seasonal graph appeared to have a regular pattern and there did not appear to be a pattern in the irregular graph, suggesting only white noise remained in the irregular component of model 9.

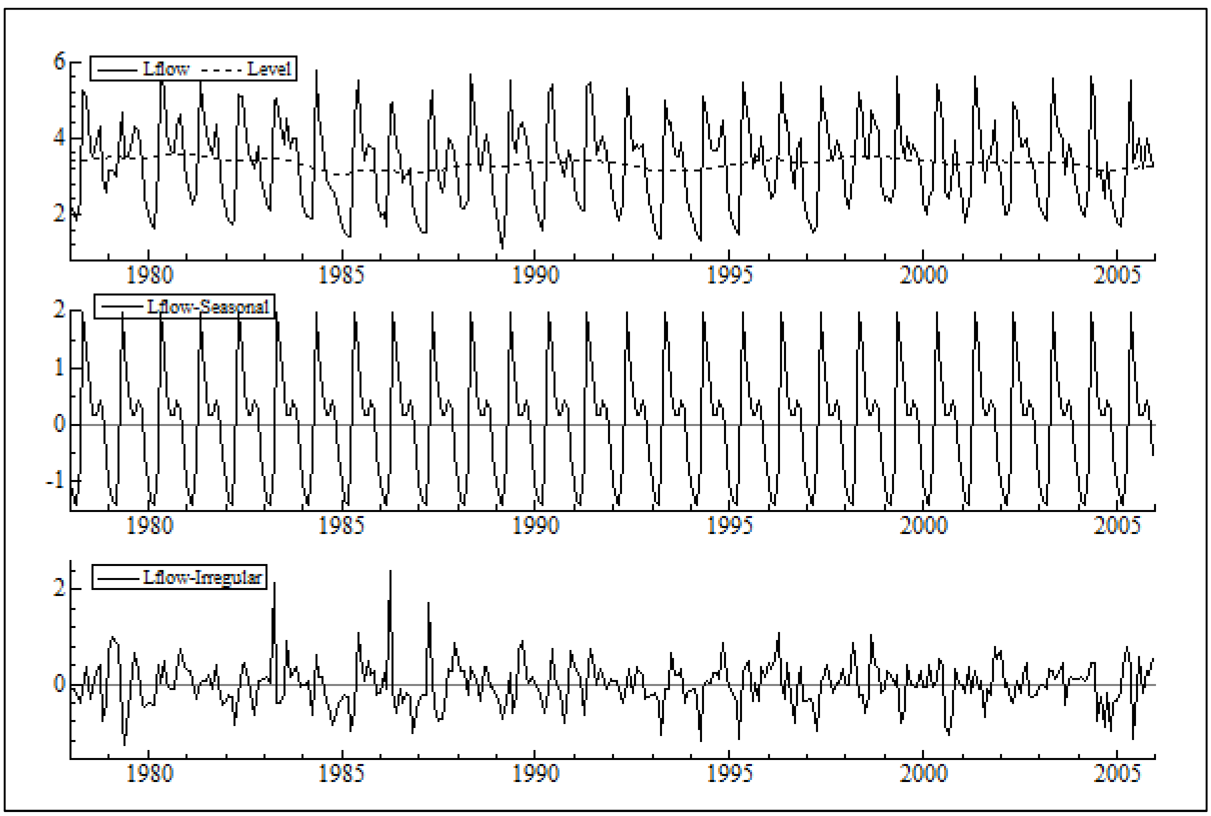


Figure 4-73: Level, Seasonal and Irregular Component Graphics for Alexis River SSTS Model 9

An examination of the diagnostic tests from Table 4-28, indicates that two of the three tests for independence, namely r(1) and Q(q,q-p), do not satisfy the assumption of independent residuals. In addition, the normality assumption is not met, however, the assumption of homoscedasticity is met. A review of the ACF plot for the model 9 residuals, Figure 4-74, shows there was still a strong lag 1 correlation in the residuals.

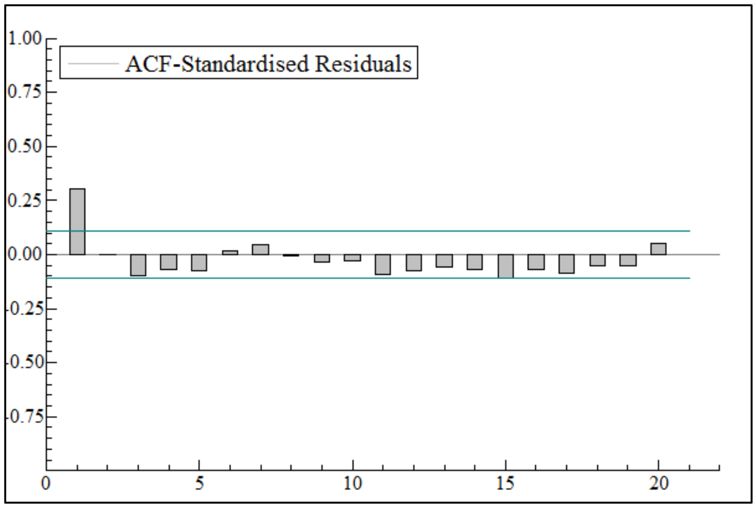


Figure 4-74: ACF Plot for Alexis River Model 9 Residuals

Model 9, which included stochastic level and deterministic seasonal components had the lowest AIC value but did not satisfy the assumption of independence as evidenced by the above ACF plot and the r(1) and Q diagnostic tests. In fact, none of the 9 models developed satisfied the independence assumption. The above plot displays the same lag 1 correlation as would be expected when reviewing ACF plots for ARMA modelling. Since the SSTS STAMP software includes the ability to add an AR1 component to the SSTS model, two more models were developed: one with stochastic level, stochastic slope, stochastic seasonal, AR1 and irregular components (model 10) and one with stochastic level, deterministic seasonal, AR1 and irregular components (model 11). The results of the diagnostic tests and AIC value are in Table 4-29.

Table 4-29: Diagnostic Tests for Alexis River SSTS Models 10 and 11

Model #	Test statistic	Value	Critical Value	Assumption
10	r(1)	0.020	0.109	Met
	r(q)	-0.046	0.109	Met
	Q(q,q-p)	13.229	30.144	Met
	H	1.774	1.984	Met
	N	34.537	5.991	Not met
	AIC	-1.372		
11	r(1)	0.016	0.109	Met
	r(q)	-0.048	0.109	Met
	Q(q,q-p)	13.131	31.410	Met
	H	1.822	1.984	Met
	N	29.069	5.991	Not met
	AIC	-1.389		

For models 10 and 11, the AIC values of -1.3273 and -1.3885, were lower than that for model 9 at -1.246. The assumption of independence was met for all three diagnostic tests and the homoscedasticity assumption was met. Only the assumption of normally distributed residuals was not met, however, this assumption is the least important assumption of the diagnostic tests (Commandeur and Koopman, 2007). Model 11 was selected for further review since it had the lowest AIC value of all the models. A review of the ACF plot for model 11, Figure 4-75, also showed that the lag 1 correlation had been addressed with this model.

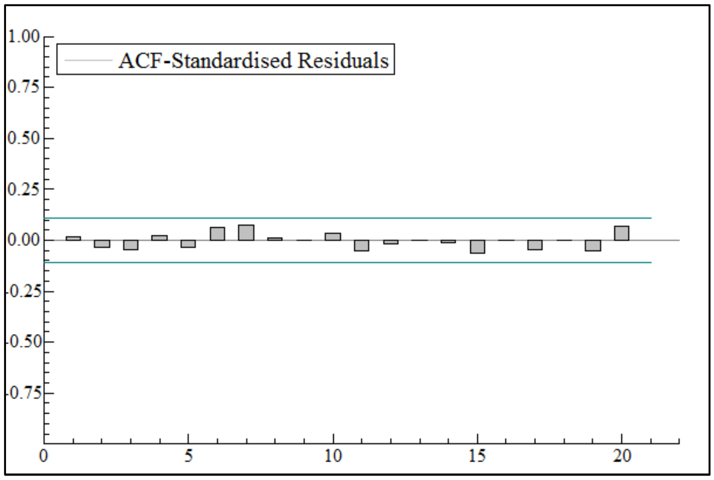


Figure 4-75: ACF plot for Alexis River model 11 residuals

Although Model 11 was the best fit for the Alexis data set, Model 9 was used to review accuracy of the model since this model is the best fitting SSTS model without the AR1 component. To verify model accuracy, the fitted values were compared to the actual data for record from 1978 to 2005. As shown in Figure 4-76 below for 1997-2005, the fitted function models the actual values fairly well.

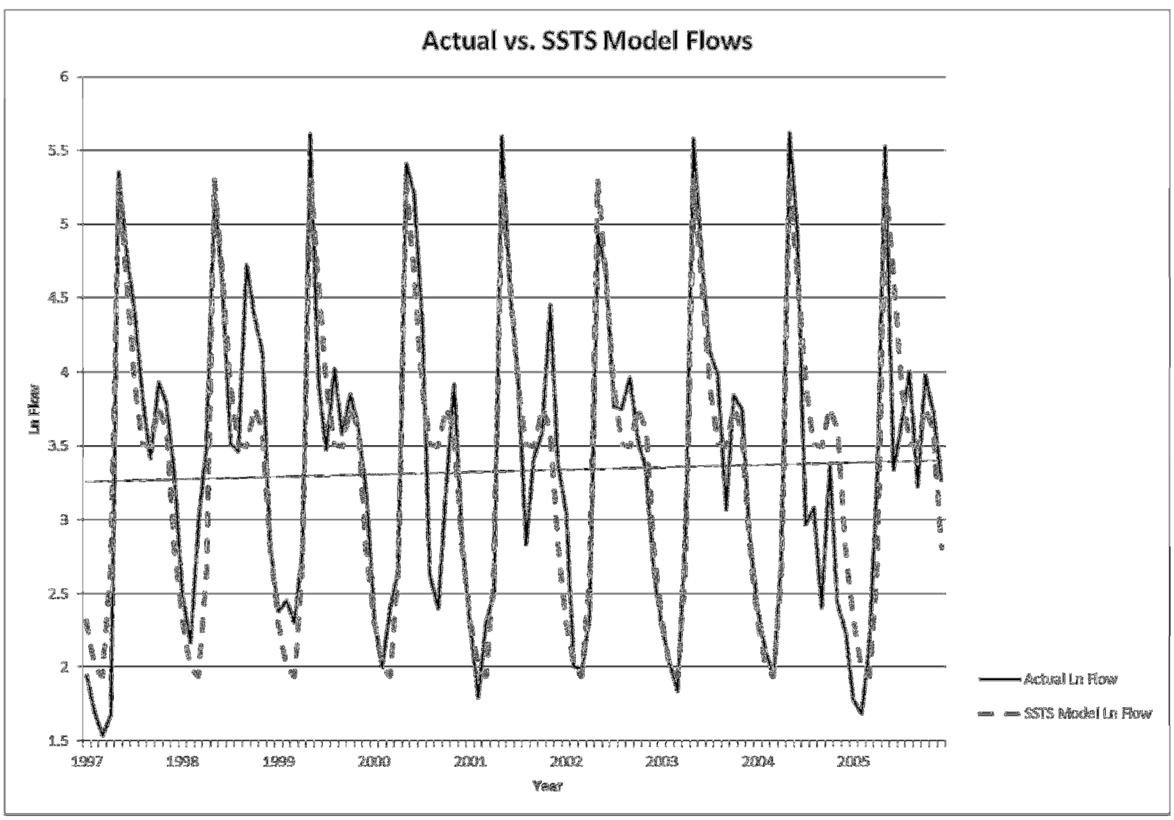


Figure 4-76: Actual Flows versus SSTS Model 9 Flows for Alexis River

The error between the fitted function values and the actual values was calculated using NSE, MSD and MAPE. The error results are provided in Table 4-30. A relatively low percentage error for MAPE as well as an NSE coefficient close to 1 indicates that the model is a good fit for the actual data set.

Table 4-30: Calculated Error between SSTS Fitted Function and Actual Flow Values for Alexis River

Years in data set	**NSE Coefficient**	**MSD**	**MAPE**
1978–2005	0.82	22.55	8.548%

4.3.1.2 Forecasting

The SSTS model was used to forecast five years of flow values, from 2006 to 2010. These correspond to the years that were removed from the original data set prior to model development. The forecast values for these years were compared to the actual flow data collected during the same time period.

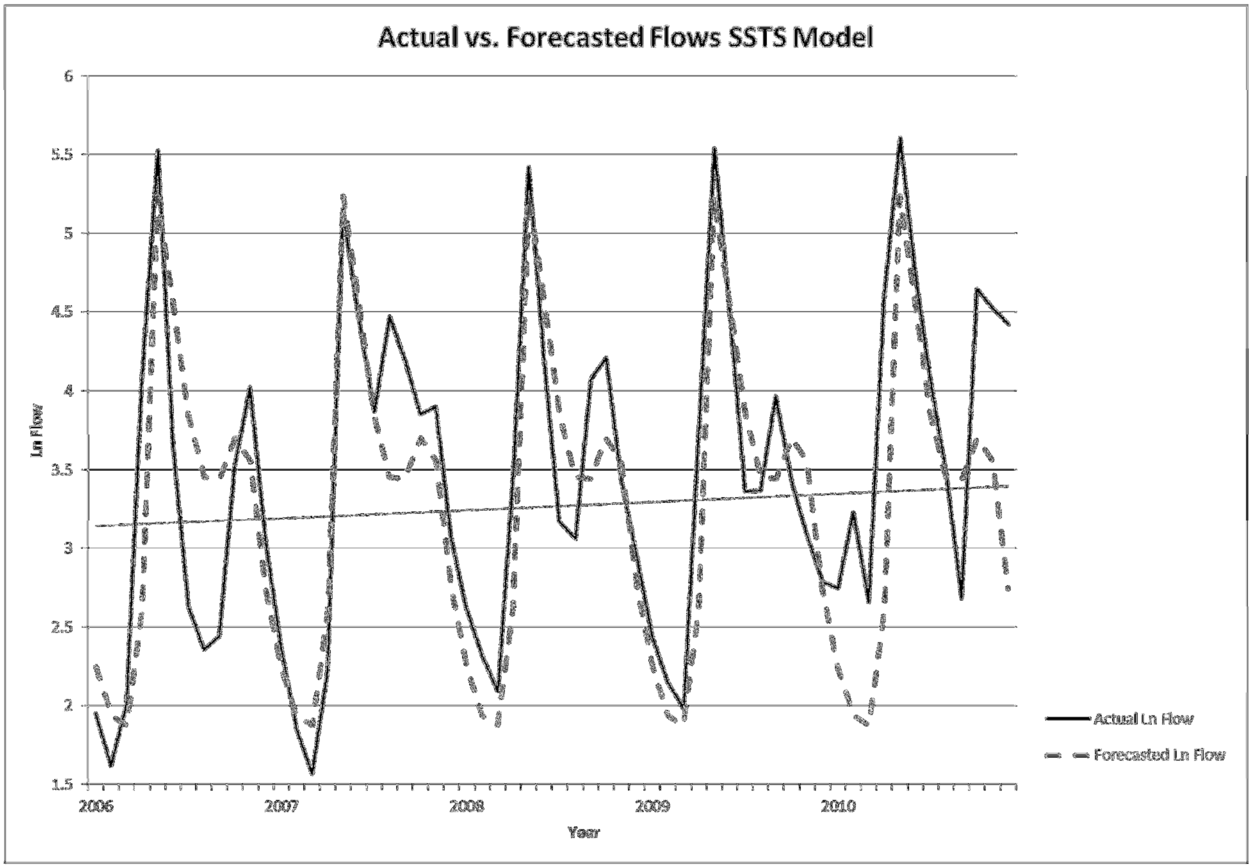


Figure 4-77: SSTS Model Forecast for Ln Alexis Flows (2006-2010)

As shown in Figure 4-77, the SSTS model appears to do a good job of predicting the first four years (2006 to 2009) but did not fit the actual data well from Nov/Dec 2009 to March 2010 as well as July to Dec 2010. A closer look at the actual data suggested that the peak patterns that exist in the data pre-2007 do not appear to be replicated between the end of 2009 and end of 2010, even though the pattern was replicated between 2006 and 2009. As was done for the ARMA methodology, three methods were used to validate the predictions: Nash–Sutcliffe efficiency, Mean Squared Deviation and Median Absolute Percentage Error. Since the actual data for late 2009–2010 did not appear to have a pattern consistent with previous years' data, the data set was segregated to specifically look at the data with and without 2009 and 2010 years. Table 4-31 shows the error calculation results.

Table 4-31: Validation Results for SSTS Analysis of Alexis River

Years in Review Set	NSE Coefficient	MSD	MAPE
Full Set (1978–2010)	0.79	25.76	8.99%
2006–2010	0.60	43.75	11.88%
2006–2008	0.70	33.33	11.88%
2009–2010	0.45	59.39	11.65%
1978–2006	0.81	23.87	8.83%

As shown in the table, the predicted flows between 2006 and 2010 show a NSE coefficient value of 0.60 which is less efficient than the full data set at 0.78. As noted for the ARMA model, a graphical review of Figure 4-77 indicates that actual flows between the winter of 2009 and winter 2010 did not appear to display the same pattern as historical flows had shown; primarily there were higher actual flows during the winter period than observed historically. As a result of this difference in pattern, the error calculations were completed separately for the 2006-2008 flows and the 2009-2010 flows. The 2006–2008 predicted flows better match the actual flow data with a NSE coefficient of 0.699 which is closer to 1, however, the 2009-2010 predicted flows do not match the actual flow data very well with a coefficient of 0.45.

Similar results were found when the MSD was calculated for the same sets of flows. A review of the three error calculations together suggest there is a larger error between predicted and actual flows in the 2009-2010 data group and the smallest error is found in the 2006-2008 predicted flows. Based on these results, the SSTS model 9 can predict short term forecasts with reasonable accuracy.

4.3.2 Ugjoktok River

As previously noted in Section 4.1.2, Ugjoktok River flow data approximately match a lognormal distribution and therefore, the Log-transformed data set was used in the SSTS analysis. Like the analysis completed using the Box Jenkins method, the data set used for SSTS model development did not include the last five years of the flow record, the data range used was from 1979 to 2005. The flows from 2006 to 2010 were reserved to compare against the forecasted values and calculate accuracy of SSTS model prediction.

4.3.2.1 Model

A number of models were developed for Ugjoktok River. Like the analysis for Alexis River, the diagnostics, graphs and AIC value were reviewed for each model with the best fitting model having diagnostic tests within critical values and the lowest AIC. In addition, the variance disturbance values in the model results provided some direction as to whether the component disturbance was deterministic or stochastic.

Similar to Alexis River, when a SSTS model with a stochastic level and an irregular component was developed, a seasonal pattern was detected in the ACF plot. This sine wave display in the plot indicated that the model required a seasonal component in the SSTS model.

A number of combinations of components were modelled and the diagnostics as well as graphics were examined. Table 4-32 outlines the component combinations for the developed models.

Table 4-32: SSTS Model Combinations for Ugjoktok River

Model #	**level**		**slope**		**seasonal**		**irregular**
	fixed	**stochastic**	**fixed**	**stochastic**	**fixed**	**stochastic**	
1	X						X
2		X					X
3		X		X		X	X
4		X			X		X
5		X				X	X

For each of the above noted models, the diagnostics were separately reviewed. The critical values were calculated in a spreadsheet and compared against the model results. Table 4-33 gives a summary of the critical values calculated for each diagnostic test along with the AIC calculation for each developed model.

Table 4-33: Diagnostic Tests for Ugjoktok River SSTS Models

	Model #	**1**	**2**	**3**	**4**	**5**
r(1)	Result value	0.640	0.147	0.215	0.162	0.162
	Critical value	0.111	0.111	0.111	0.111	0.111
	assumption	Not met	Not met	Not met	Not met	Not met
r(q)	Result value	0.813	0.690	0.046	0.011	0.011
	Critical value	0.111	0.111	0.111	0.111	0.111
	assumption	Not met	Not met	met	met	met
Q(q,q-p)	Result value	1571.700	524.080	42.190	28.750	28.750
	Critical value	35.172	35.172	32.671	33.924	33.924
	assumption	Not met	Not met	Not met	met	met
H(h)	Result value	1.109	1.012	1.338	1.082	1.082
	Critical value	1.984	1.984	1.984	1.984	1.984
	assumption	Met	Met	Met	Met	Met
N	Result value	15.614	618.310	91.943	115.190	115.190
	Critical value	5.991	5.991	5.991	5.991	5.991
	assumption	Not met	Not met	Not met	Not met	Not met
AIC		0.407	0.082	-1.569	-1.645	-1.645
convergence		Very strong	Very strong	weak	weak	weak
Variance of disturbance	Level		1.076	0	0.0006	0.0006
	Slope			0		
	Seasonal			0		0

A review of Table 4-33 shows that models 4 and 5 resulted in the lowest AIC value. As identified in Table 4-32, model 4 includes stochastic level, deterministic seasonal and irregular components while model 5 includes stochastic level, stochastic seasonal and irregular components. An examination of the variance of disturbance results indicated that for model 5, the variance of disturbance for the seasonal component was zero. This result suggests that the seasonal component, though selected in this model as stochastic, was in fact deterministic, meaning model 4 and model 5 were actually the same model with a deterministic seasonal component. Based on AIC and variance review, model 4, which modelled a deterministic seasonal component, was potentially the best fitting model. This model included the same components as Alexis River, a deterministic seasonal, stochastic level and irregular components.

An evaluation of the component graphics for model 4, as shown in Figure 4-78, indicated that the level equation was stochastic as evidenced by the undulating dashed line in the top graph. The seasonal graph appeared to have a regular pattern and there did not appear to be a pattern in the irregular graph, suggesting only white noise remained in the irregular component of model 4.

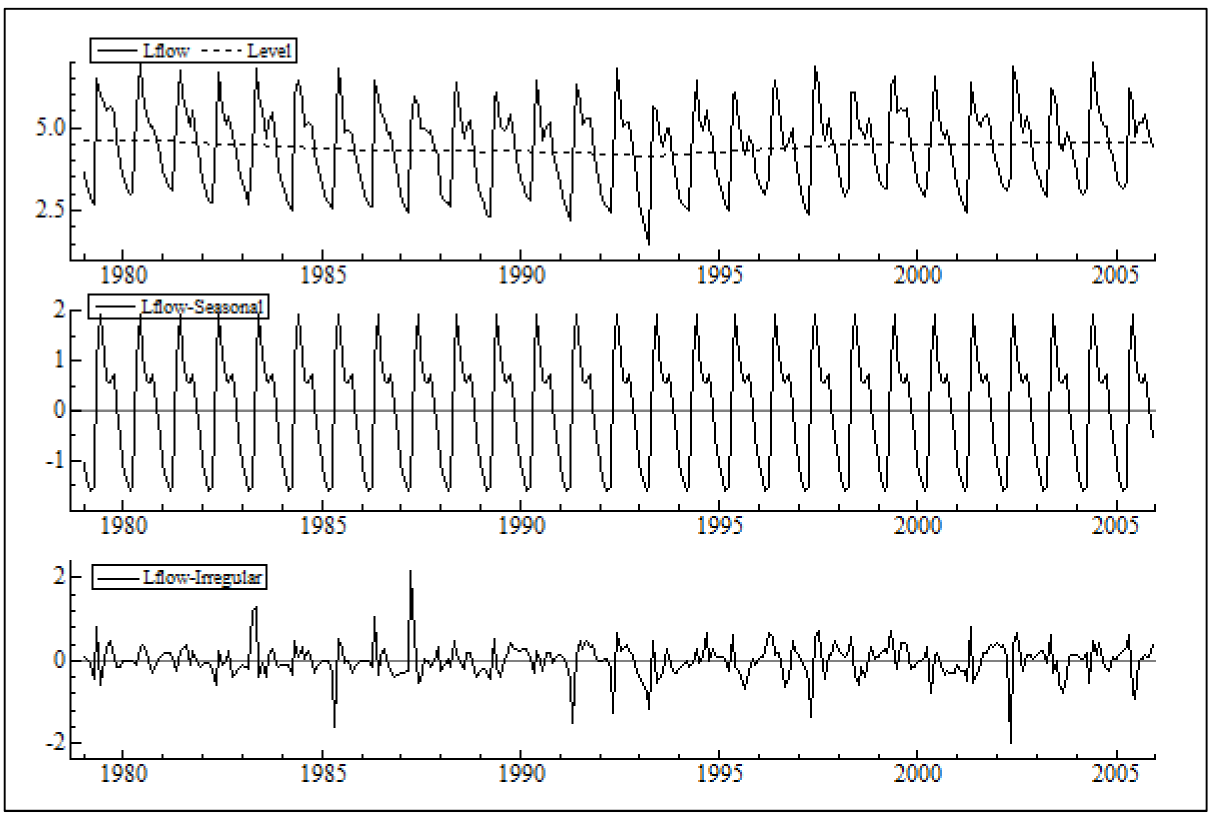


Figure 4-78: Level, Seasonal and Irregular Component Graphics for Ugjoktok River SSTS Model 4

An examination of the diagnostic tests from Table 4-33, indicated that one of the three test for independence, namely r(1), did not satisfy the assumption of independent residuals. In addition, the normality assumption was not met, however, the assumption of homoscedasticity was met. A review of the ACF plot for the model 4 residuals, Figure 4-79, showed there was still a lag 1 correlation in the residuals.

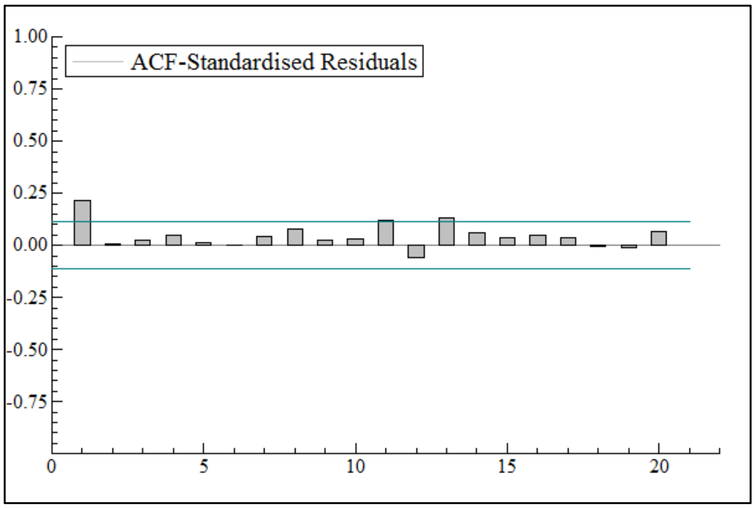


Figure 4-79: ACF Plot for Ugjoktok River Model 4 Residuals

Model 4, which included stochastic level and deterministic seasonal components had the lowest AIC value but did not satisfy the assumption of independence as evidenced by the above ACF plot and the r(1) diagnostic test. Similar to the SSTS analysis for Alexis River, none of the 5 models developed satisfied the independence assumption. The above plot displays the same lag 1 correlation as would be expected when reviewing ACF plots for ARMA modelling. One additional model, model 6, was developed including stochastic level, stochastic slope, stochastic seasonal, AR1 and irregular components. The results of the diagnostic tests and AIC value are in Table 4-34.

Table 4-34: Diagnostic Tests for Ugjoktok River SSTS Model 6

Model #	Test statistic	Value	Critical Value	Assumption
6	r(1)	0.014	0.111	Met
	r(q)	0.028	0.111	Met
	Q(q,q-p)	26.146	31.410	Met
	H	1.050	1.984	Met
	N	124.540	5.991	Not met
	AIC	-1.674		

For model 6, the AIC value of -1.674 is marginally lower than that for model 4 at -1.645. The assumption of independence was met for all three diagnostic tests and the homoscedasticity assumption was met. Only the assumption of normally distributed residuals was not met, however, this assumption is the least important assumption of diagnostic tests (Commandeur and Koopman, 2007). A review of the ACF plot, Figure 4-80, also showed that the lag 1 correlation had been addressed with this model.

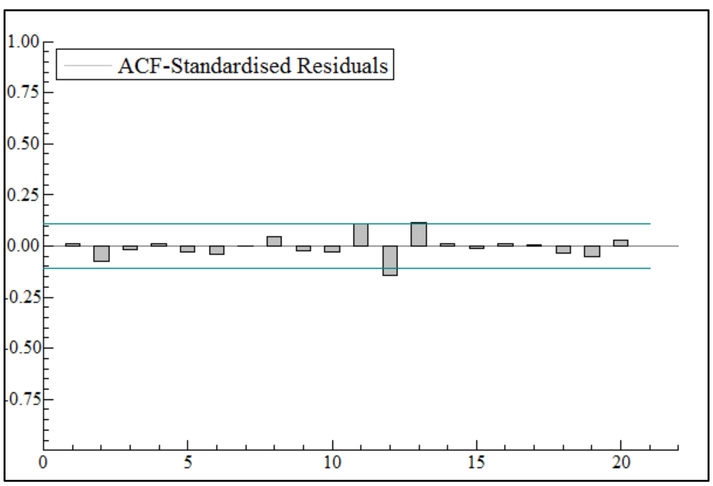


Figure 4-80: ACF Plot for Ugjoktok River Model 6 Residuals

Although Model 6 was the best fit to the Ugjoktok data set, Model 4 was used to review accuracy of the model since this was the best fitting SSTS model without the AR1 component. To verify model accuracy, the fitted values were compared to the actual data for record from 1979 to 2005. As shown in Figure 4-81 below, the fitted function models the actual values fairly well.

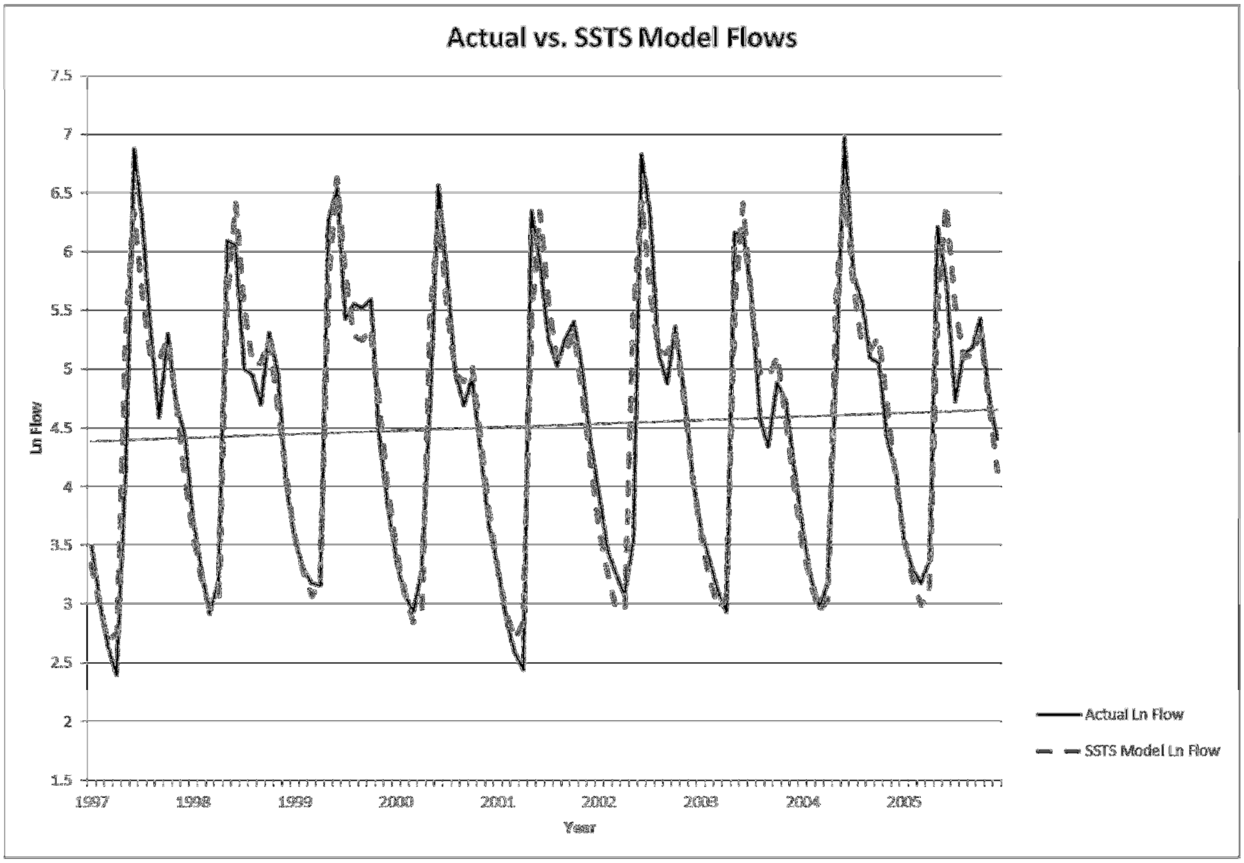


Figure 4-81: Actual Flows versus SSTS Model 4 Flows for Ugjoktok River

The error between the fitted function values and the actual values was calculated using NSE, MSD and MAPE. The error results are provided in Table 4-35. A low percentage error for MAPE as well as an NSE coefficient close to 1 indicates that the model is a good fit for the actual data set.

Table 4-35: Calculated Error between SSTS Fitted Function and Actual Flow Values for Ugjoktok River

Years in Review Set	NSE Coefficient	MSD	MAPE
1979–2005	0.92	12.45	3.91%

4.3.2.2 Forecasting

The SSTS model was used to forecast five years of flow values, from 2006 to 2010. These correspond to the years that were removed from the original data set prior to model development. The forecast values for these years were compared to the actual flow data collected during the same time period.

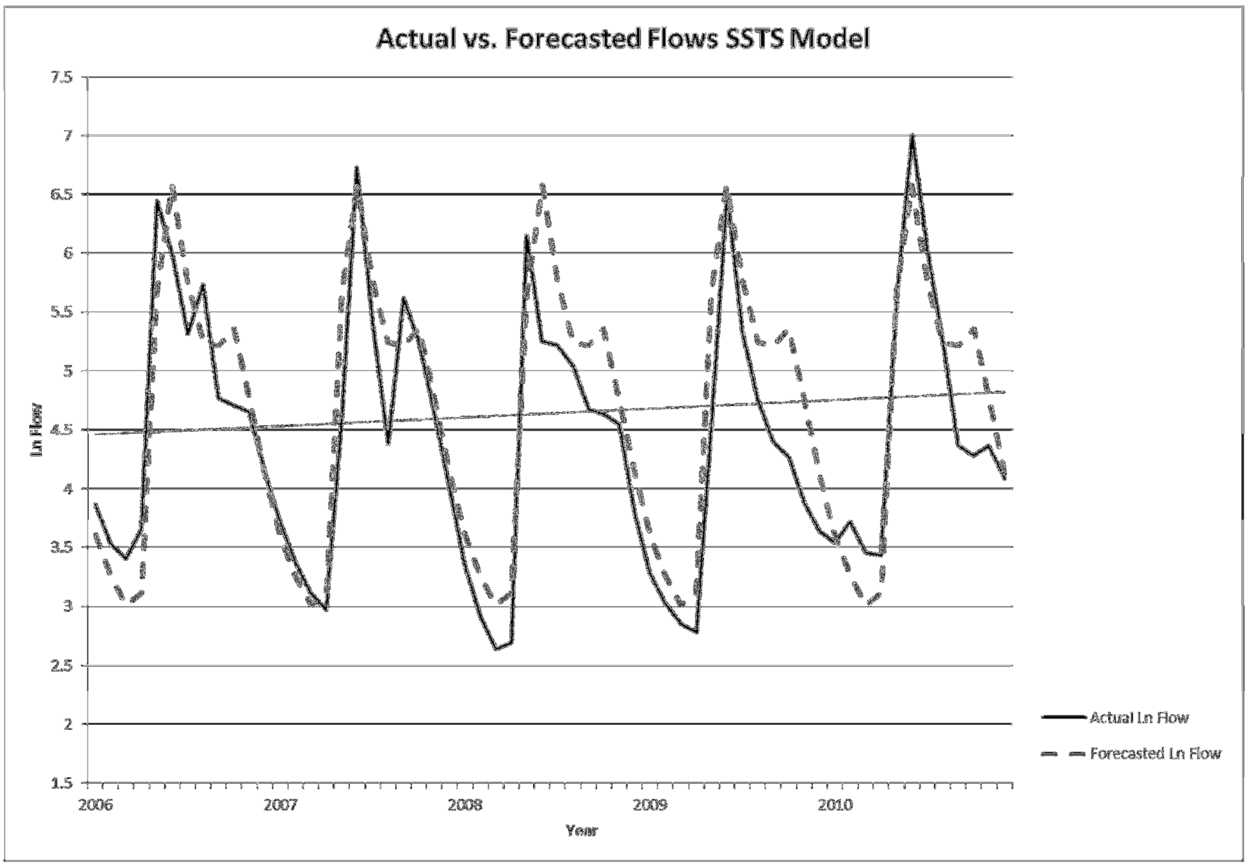


Figure 4-82: SSTS Forecast for Ugjoktok River Flows (2006-2010)

As shown in Figure 4-82, the SSTS model appeared to do a reasonably good job of predicting the first two years (2006 to mid 2008) but did not fit the actual data well, generally in the late summer to early winter for the remaining three years. The September peak pattern that exists in the data pre-2006 did not appear to be replicated between 2008 and 2010, even though the pattern was replicated in 2006 and 2007. As was done for the ARMA methodology, three methods were used to validate the predictions:

Nash–Sutcliffe efficiency, Mean Squared Deviation and Median Absolute Percentage Error. The years were grouped to be consistent with the Alexis River error calculation groupings. In addition, the 2006-2007 years were separately reviewed since the forecasts for these two years most closely represented the actual values observed. Table 4-36 shows the error calculation results.

Table 4-36: Validation Results for SSTS Analysis of Ugjoktok River

Years in Review Set	NSE Coefficient	MSD	MAPE
Full Set (1979–2010)	0.90	14.77	4.29%
2006–2010	0.77	27.30	8.66%
2006-2007	0.82	20.71	7.02%
2006–2008	0.79	24.36	8.14%
2009–2010	0.74	31.71	9.51%

As shown in the table, the predicted flows between 2006 and 2010 show a NSE coefficient value of 0.77 which is less efficient than the full data set at 0.90. As previously discussed, error calculations were completed separately for the 2006-2007, 2006-2008 and 2009-2010 records, as a result of a difference in pattern between the actual observed values and the forecasted values. Upon review of the NSE coefficient results, the 2006-2007 predicted flows better matched the actual flow data for the same period, however, the 2006-2008 and 2009-2010 predicted flows did not match the actual flow data sets as well, with coefficients of 0.79 and 0.74 respectively.

Similar results were found when the MAPE and MSD were calculated for the same sets of flows. A review of the three error calculations together suggest there is a larger error between predicted and actual flows in the 2009-2010 data group and the smallest error is found in the 2006-2007 predicted flows. Based on these results, the SSTS Model 4 can predict short term flows with reasonable accuracy.

4.3.3 Romaine River

As previously noted in Section 4.1.3, Romaine River streamflow data approximately match a lognormal distribution and as such, the Log-transformed data set was used in the SSTS analysis. Like the analysis completed using the Box Jenkins method, the data set used for SSTS model development did not include the last five years of the flow record; the data range used was from 1960 to 2005. The flows from 2006 to 2010 were reserved to compare against the forecasted values and calculate accuracy of SSTS model prediction.

4.3.3.1 Model

A number of models were developed for the Romaine River. Like the analyses for the Alexis and Ugjoktok Rivers, the diagnostics, graphs and AIC value were reviewed for each model with the best fitting model having diagnostic tests within critical values and the lowest AIC. In addition, the variance disturbance values in the model results provided some direction as to whether the component disturbance was deterministic or stochastic.

Similar to Alexis and Ugjoktok Rivers, when an SSTS model with a stochastic level and an irregular component was developed, a seasonal pattern was detected in the ACF plot. This seasonality indicated that the model required a seasonal component in the SSTS model. A number of combinations of components were modelled and the diagnostics as well as graphics were examined.

Table 4-37 outlines the component combinations for the developed models.

Table 4-37: SSTS Model Combinations for the Romaine River

Model #	**level**		**slope**		**seasonal**		**irregular**
	fixed	**stochastic**	**fixed**	**stochastic**	**fixed**	**stochastic**	
1	X						X
2		X		X		X	X
3		X	X		X		X
4		X				X	X
5		X			X		X

For each of the above noted models, the diagnostics were separately reviewed. The critical values were calculated in a spreadsheet and compared against the model results. Table 4-38 gives a summary of the critical values calculated for each diagnostic test along with the AIC calculation for each developed model.

Table 4-38: Diagnostic Tests for the Romaine River SSTS Models

	Model #	**1**	**2**	**3**	**4**	**5**
r(1)	Result value	0.610	0.316	0.304	0.322	0.309
	Critical value	0.086	0.086	0.086	0.086	0.086
	assumption	Not met	Not met	Not met	Not met	Not met
r(q)	Result value	0.825	0.044	0.041	-0.017	0.036
	Critical value	0.086	0.086	0.086	0.086	0.086
	assumption	Not met	met	met	met	met
Q(q,q-p)	Result value	2143.100	100.610	88.100	99.110	86.920
	Critical value	35.172	33.924	33.924	33.924	33.924
	assumption	Not met	Not met	Not met	Not met	Not met
H(h)	Result value	1.095	1.110	1.028	1.099	1.018
	Critical value	1.984	1.984	1.984	1.984	1.984
	assumption	Met	Met	Met	Met	Met
N	Result value	26.870	12.450	12.500	12.030	12.370
	Critical value	5.991	5.991	5.991	5.991	5.991
	assumption	Not met	Not met	Not met	Not met	Not met
AIC		-0.321	-2.152	-2.174	-2.157	-2.178
convergence		Very strong	Very strong	Very strong	strong	Very strong
Variance of disturbance	Level		0.0009	0.0007	0.0007	0.0006
	Slope		0			
	Seasonal		0			

A review of Table 4-38 shows that model 5 which included stochastic level, deterministic seasonal and irregular components, had the lowest AIC value. Based on AIC and variance review, model 5 was potentially the best fitting model. This model included the same components as Alexis and Ugjoktok Rivers, a deterministic seasonal, stochastic level and irregular components.

An evaluation of the component graphics for model 5, as shown in Figure 4-83, indicated that the level equation was stochastic and evidenced by the undulating dashed line in the top graph. The seasonal graph appeared to have a regular pattern and there did not appear to be a pattern in the irregular graph, suggesting only white noise remained in the irregular component of model 5.

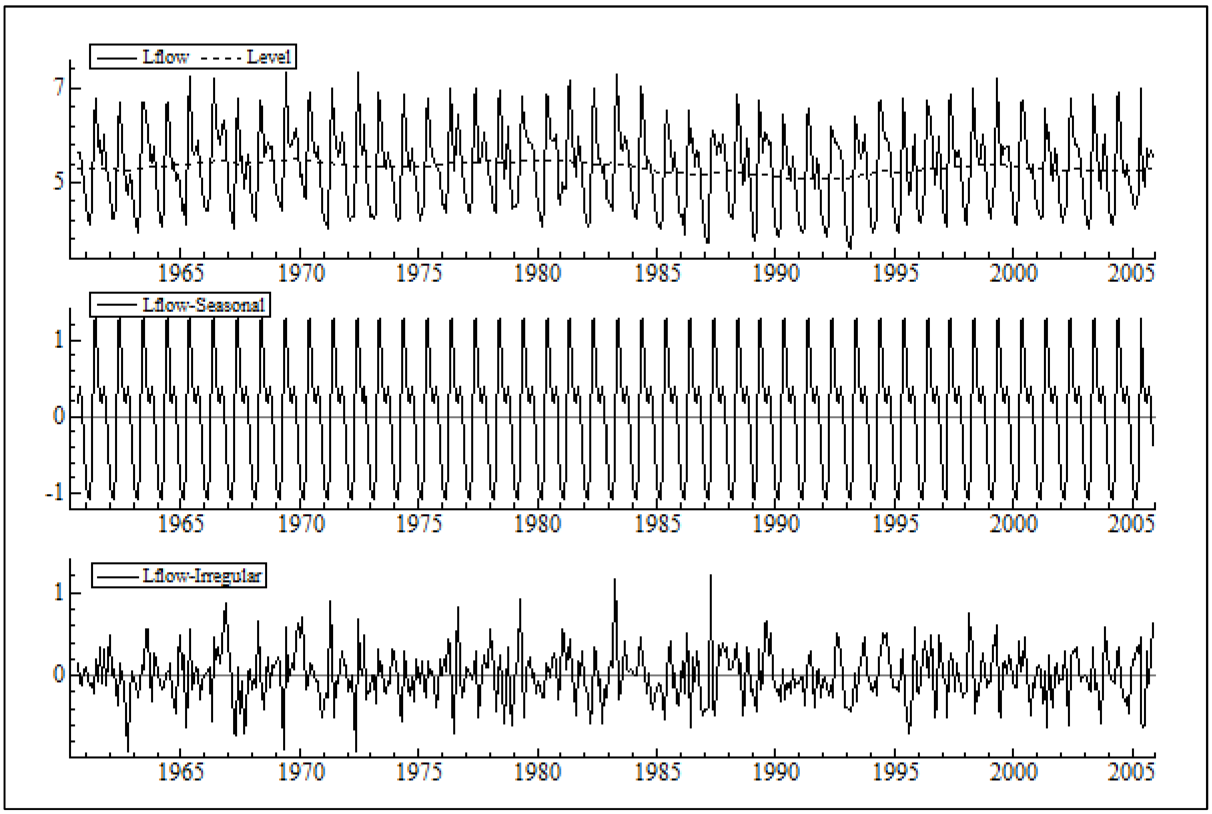


Figure 4-83: Level, Seasonal and Irregular Component Graphics for Romaine River SSTS Model 5

An examination of the diagnostic tests from Table 4-38, indicated that two of the three tests for independence, namely r(1) and Q(q,q-p), did not satisfy the assumption of independent residuals. In addition, the normality assumption was not met, however, the assumption of homoscedasticity was met. A review of the ACF plot for the model 5 residuals, Figure 4-84, showed there was still a strong lag 1 correlation in the residuals.

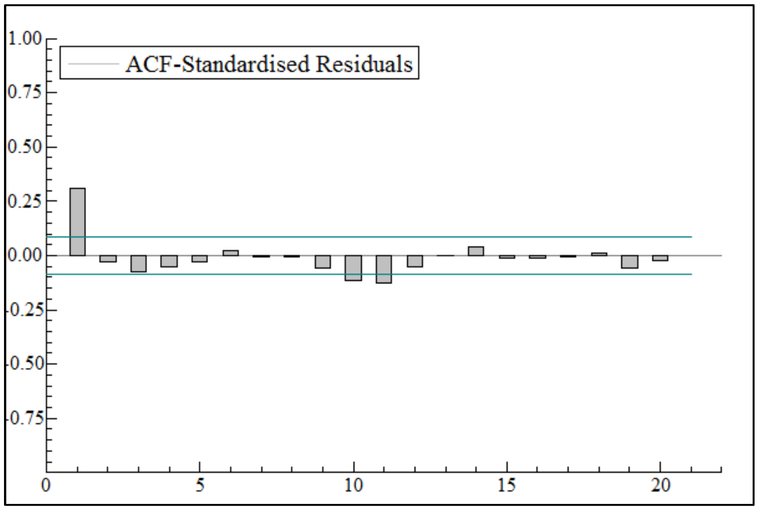


Figure 4-84: ACF Plot for Romaine River Model 5 Residuals

Model 5, which included stochastic level and deterministic seasonal components, had the lowest AIC value but did not satisfy the assumption of independence as evidenced by the above ACF plot and the r(1) and Q diagnostic tests. Similar to the SSTS analysis for Alexis River and Ugjoktok River, none of the initial models developed satisfied the independence assumption. The above plot displays the same lag 1 correlation as would be expected when reviewing ACF plots for ARMA modelling. One additional model for the Romaine River, model 6, was developed including stochastic level, stochastic slope, deterministic seasonal, AR1 and irregular components. The results of the diagnostic tests and AIC value are in Table 4-39.

Table 4-39: Diagnostic Tests for Romaine River SSTS Model 6

Model #	Test Statistic	Value	Critical Value	Assumption
6	r(1)	0.038	0.086	Met
	r(q)	0.042	0.086	Met
	Q(q,q-p)	32.447	33.924	Met
	H	1.029	1.984	Met
	N	19.940	5.991	Not met
	AIC	-2.293		

For model 6, the AIC value of -2.293 was marginally lower than that for model 5 at -2.178. The assumption of independence was met for all three diagnostic tests and the homoscedasticity assumption was met. Only the assumption of normally distributed residuals was not met, however, this assumption is the least important assumption of diagnostic tests (Commandeur and Koopman, 2007). A review of the ACF plot, Figure 4-85, also shows that the lag 1 correlation has been addressed with this model.

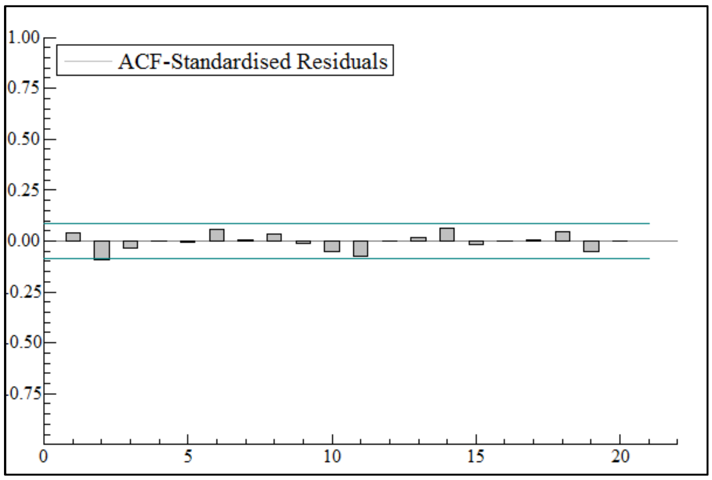


Figure 4-85: ACF Plot for Romaine River Model 5 Residuals

Although Model 6 was the best fit for the Romaine data set, model 5 was used to review accuracy of the model since this was the best fitting SSTS model without the AR1 component. To verify model accuracy, the fitted values were compared to the actual data for record from 1960 to 2005. As shown in Figure 4-86 below, the fitted function models the actual values fairly well.

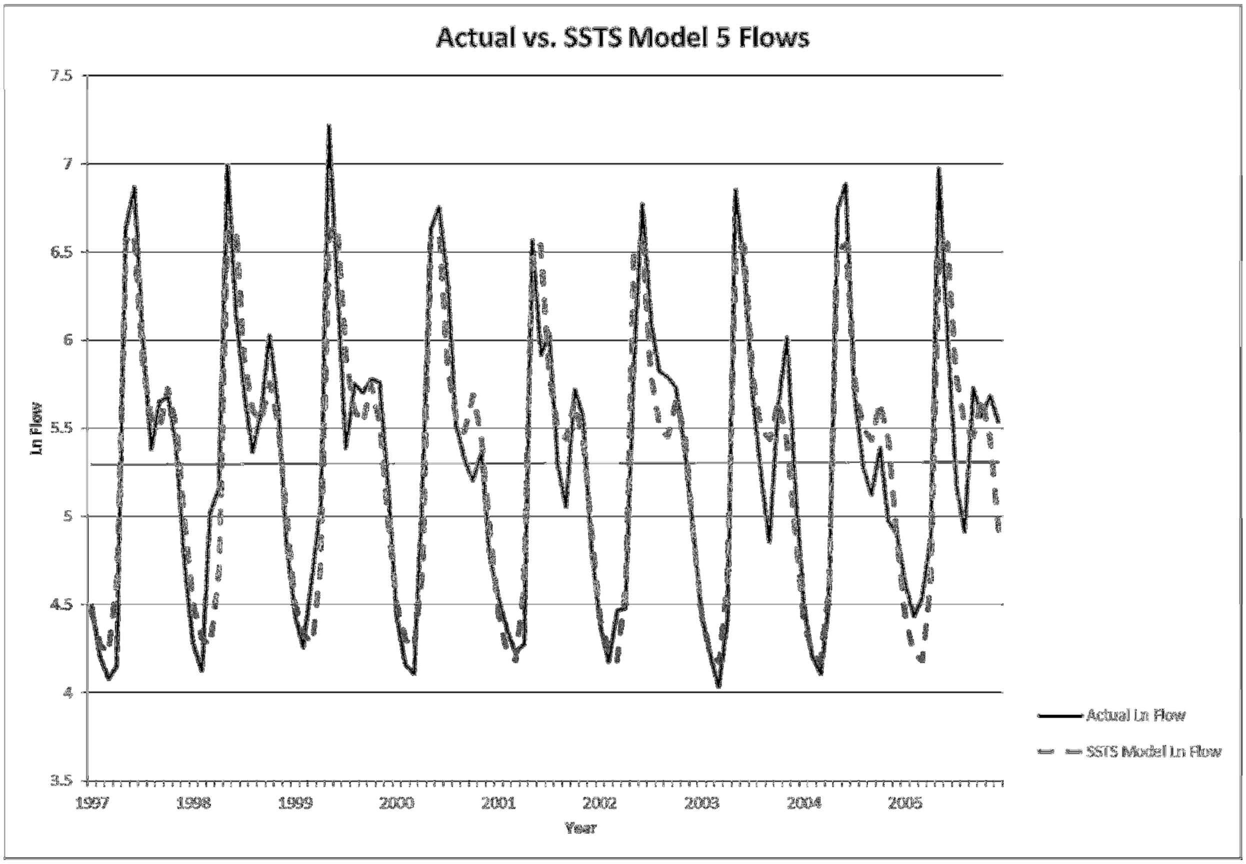


Figure 4-86: Actual Flows versus SSTS Model 5 Flows for Romaine River

The error between the fitted function values and the actual values was calculated using NSE, MSD and MAPE. The error results are provided in Table 4-40. A low percentage error for MAPE as well as an NSE coefficient close to 1 indicates that the model is a good fit for the actual data set.

Table 4-40: Calculated Error between SSTS Modelled Flows and Actual Flows for the Romaine River

Years in data set	NSE Coefficient	MSD	MAPE
1960–2005	0.87	9.63	3.61%

4.3.3.2 Forecasting

The SSTS model was used to forecast five years of flow values, from 2006 to 2010. These correspond to the years that were removed from the original data set to develop the model. The forecast values for these years were compared to the actual flow data collected during the same time period.

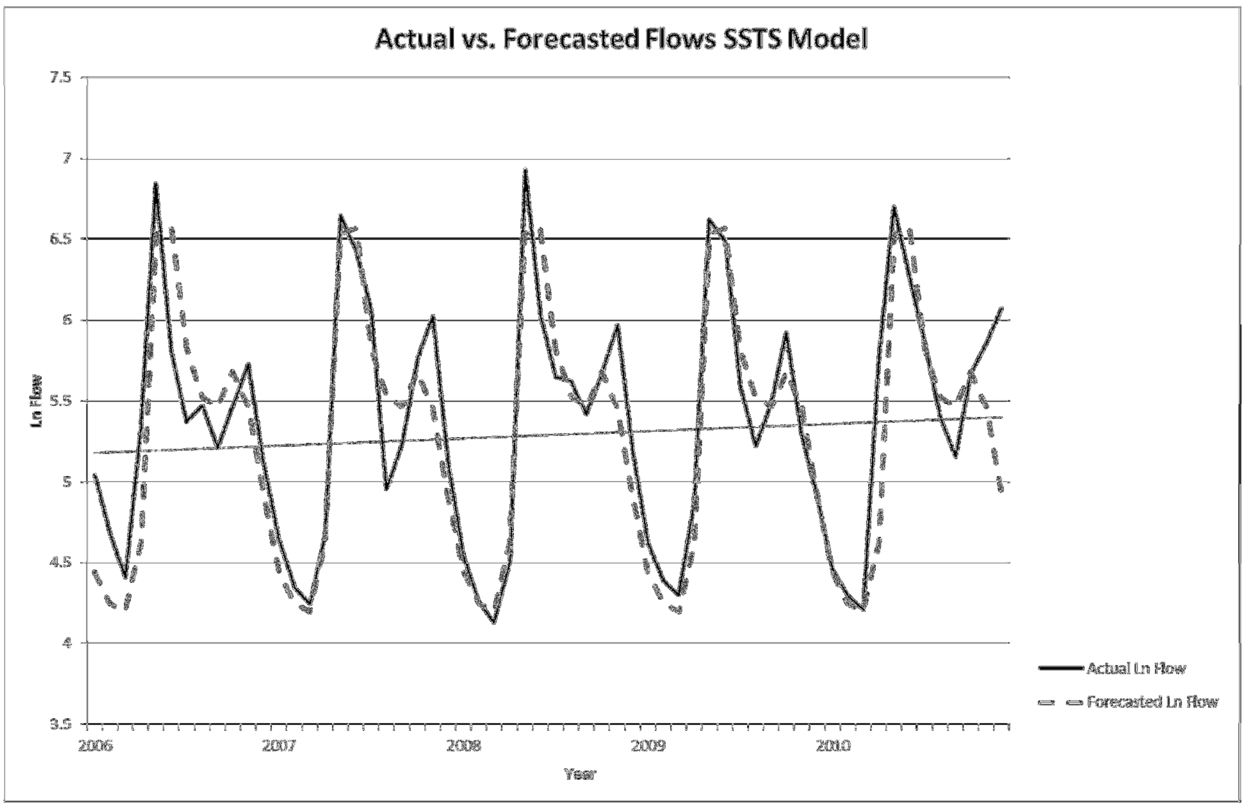


Figure 4-87: SSTS Forecast for Romaine River Flows (2006-2010)

As shown in Figure 4-87, the SSTS model appears to do a reasonably good job of predicting all five years with the exception of the last three months of 2010. To verify the prediction accuracy of the model and as was done for the ARMA methodology, three methods were used to validate the predictions: Nash–Sutcliffe efficiency, Mean Squared Deviation and Median Absolute Percentage Error. Years were grouped to match the Alexis and Ugjoktok River groupings for comparison purposes. Table 4-41 shows the error calculation results.

Table 4-41: Validation Results for SSTS Analysis of Romaine River

Years in Review Set	NSE Coefficient	MSD	MAPE
Full Set (1960–2010)	0.86	9.88	3.62%
2006–2010	0.78	12.13	3.89%
2006–2008	0.80	10.90	4.06%
2009–2010	0.76	13.97	2.70%

As shown in the table, the predicted flows between 2006 and 2010 showed a NSE coefficient value of 0.78 which is less efficient than the full data set at 0.86. Figure 4-87 suggests that SSTS forecast values closely model the actual flows between 2006 and 2010. The NSE coefficient values as well as the MSD values were consistent and showed the 2006–2008 predicted flows better match the actual data than do the values for 2006-2010 or 2009-2010. Like the deseasonalized ARMA model results, these findings are not supported using MAPE, with the MAPE results suggesting the forecasts were not as closely modelled for the 2006-2008 period as to the 2009-2010 period. It should be noted, however, that 2.70% and 4.06% error for the 2009-2010 and 2006-2008 records respectively, is a small margin of error. Based on these results, it was determined that SSTS model 5 for the Romaine River could predict short term forecasts with good accuracy.

Chapter 5 Comparison of Modelling Methods

The three rivers were modelled using two time series methods: deseasonalized ARMA analysis based on Box Jenkins Time Series methodology and State-Space Time Series Analysis. Following is a comparison of the results using these modelling methods for the studied rivers.

A graphical review of the fitted values against the actual values, up to the end of 2005, for both the deseasonalized ARMA method and the SSTS method indicates that both methods model the actual values reasonably well for all three rivers. Error calculations using NSE coefficient, MSD and MAPE, summarized in Table 5-1, illustrate that the SSTS analysis model, in fact, better fit the actual data set for all three rivers.

Table 5-1: Comparison of Calculated Error for Fit of Box Jenkins and SSTS Models for All Three Rivers

River	Years in Review Set	Model Method	NSE Coefficient	MSD	MAPE
Alexis	1978–2005	ARMA (Fourier)	0.78	27.86	9.93%
		SSTS	0.82	22.55	8.55%
Ugjoktok	1979-2005	ARMA (Fourier)	0.87	19.14	5.77%
		SSTS	0.92	12.45	3.91%
Romaine	1960-2005	ARMA (Fourier)	0.85	11.57	4.04%
		SSTS	0.87	9.63	3.61%

Also worth noting is the narrow range of error values between the rivers for each type of model. For the deseasonalized ARMA method, the NSE coefficient ranges from 0.78 to 0.87 and MAPE ranges from 4.04% to 9.93%. Likewise, the error between the rivers using SSTS, ranges from 0.82 to 0.92 for the NSE coefficient and 3.61% to 8.55% for MAPE. The rivers have similar ARMA and SSTS model structures resulting in a similar fit.

The preferred SSTS and deseasonalized ARMA models were used to forecast flows up to 5 years, 2006 to 2010. These five years were removed from the data set for each river prior to model development for both methods. Graphical plots comparing the actual to forecasted data show the later forecast years for all three rivers did not fit the actual data well: the peak patterns that existed in the data pre-2006 did not appear to be replicated in the later forecast years even though the pattern is replicated between 2006 and 2008. Calculated error for a number of different forecast combinations indicated that the shorter term forecast, 2006 to 2008, had the least amount of error from the actual reserved flows. Table 5-2 summarizes the comparison of the 2006-2008 forecasts for the preferred models and confirms SSTS displayed less error in the forecasted flows for the Alexis and Romaine Rivers while deseasonalized ARMA displayed less error in the forecasts for the Ugjoktok River. In fact, the error calculation results are very close and both modelling methods could be considered equally suitable for forecasting these rivers.

Table 5-2: Comparison of Calculated Error of Forecasts using Deseasonalized ARMA and SSTS Models for All Three Rivers

River	Years in Review Set	Model Method	NSE Coefficient	MSD	MAPE
Alexis	2006-2008	ARMA (Fourier)	0.70	33.46	12.58%
		SSTS	0.70	33.33	11.88%
Ugjoktok	2006-2008	ARMA (Fourier)	0.81	22.11	6.34%
		SSTS	0.79	24.36	8.14%
Romaine	2006-2008	ARMA (Fourier)	0.79	11.94	3.36%
		SSTS	0.80	10.90	4.06%

Chapter 6 Conclusions and Recommendations

Three rivers in Labrador and South-East Quebec, namely the Alexis, Ugjoktok and Romaine Rivers, were chosen for this study after completing a specific screening process based on size of drainage area upstream of hydrometric station, geographic location and basin aspect. These rivers were modelled using two time series methods: deseasonalized ARMA analysis based on Box Jenkins Time Series methodology and State-Space Time Series Analysis. These methods were specifically chosen to compare each modelling method for its ability to accurately represent the measured streamflows as well as predict future flows.

6.1 Conclusions

In drawing conclusions from this study, the relevant differences between the two types of modelling methods are described herein.

ARMA differs from SSTS as it relates to the application of non-stationary data. In order to use ARMA, the time series must be stationary. Since hydrological data display seasonality, and the data presented in this study is no exception, the original time series is not stationary. As a result, in order to analyze a hydrologic time series using ARMA, the time series must be made stationary by removing the seasonality, in this case using Fourier, and then modelling the residuals of that model using ARMA. SSTS does not require time series data to be stationary since components such as seasonality and trend are explicitly modelled using state space equations.

The ARMA approach requires missing data to be infilled. Both Ugjoktok and Romaine had missing data in the time series and in order to appropriately model the records using ARMA, the missing data needed to be filled using techniques such as LOC. SSTS does not require missing data to be infilled. The Kalman filter is used to estimate missing data points using previously observed data, much the same as it is used to predict future values. This method will also produce a larger variance estimation for the missing data points than using LOC to infill the missing points, which would be expected.

The best fitting deseasonalized ARMA method for all three rivers included a Fourier 5 spectral analysis model to represent seasonality and an AR1 to model the remaining pattern in the deseasonalized data. The best fitting SSTS model for all three rivers included stochastic level, deterministic seasonal and irregular components. Each of the three SSTS models could be improved by adding an AR1 component to the basis state equations. Both methods required an ARMA component to address the pattern in the streamflow data. This result suggests that for some hydrological time series with strong autocorrelation, SSTS produces a better fit when an ARMA component is included in the model.

The resulting models using the deseasonalized ARMA method have a similar time series structure: Fourier 5 seasonal model with an AR1 model of the residuals, regardless of basin size, geographic location (North or South of Goose Bay) and basin aspect (outlet to Labrador Sea or Gulf of St. Lawrence). The three rivers also displayed the same three seasonality periods: 12, 6 and 4 months.

The resulting models using the SSTS method have a similar time series structure including stochastic level, deterministic seasonal and irregular components, regardless of basin size, geographic location (North or South of Goose Bay) and basin aspect (outlet to Labrador Sea or Gulf of St. Lawrence).

The SSTS analysis produced marginally better fitting models for the Alexis, Ugjoktok and Romaine Rivers than the deseasonalized ARMA method. The deseasonalized ARMA method, however, was easier to diagnose compared to the SSTS method, primarily due to the level of understanding required to calculate and interpret the critical diagnostic test values.

The SSTS and deseasonalized ARMA models predicted future flows with similar levels of accuracy: the SSTS model having slightly less error for the Alexis and Romaine Rivers and the ARMA model having slightly less error for the Ujgoktok River. In general, the SSTS method was marginally better at short term monthly flow forecasts.

The SSTS model cannot be easily used for simulations, either to verify the model or to produce energy estimates like flow duration curve analysis. Simulation can be accomplished by programming a software package like Oxmetrics ssf pack, however, many engineers may not be fluent in this level of programming, resulting in the inability for many to analyze potential energy production using a flow duration curve or other methods requiring simulations from an SSTS model. Since simulation of deseasonalized ARMA models is relatively straightforward and easy to achieve, flow duration curve development and other types of energy analysis for planning of future hydroelectric projects are more easily achieved using this method.

6.2 Recommendations

The SSTS method using STAMP is a very quick way to develop a model, however the diagnostic evaluation could be improved with the inclusion of the critical values in the model results section. In addition, including a simulation feature in the program, whereby multiple simulations could developed, would make this method superior to the deseasonalized ARMA method when simulation and forecasting of stream flows is the primary goal.

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APPENDIX A

MACROS

```
GMACRO
spectrum
erase c3-c10
Note
Note Data in C1 and Index or Year in C2.
Note
name c3 'r(k)' c4 'k' c5 'w(k)' c6 'f' c7 'r x w'
name c8 'rwcos' c9 'sdf' c10 'period'
Note: Enter number of lags to use.  E.g. 24
set c50;
file "terminal";
nobs 1.
let k2=c50(1)
acf k2 c1 c3
set c4
1:k2
end
let c5=0.5*(1+cos(3.14159*c4/k2))
let c6=c4/(2*k2)
let c7=c5*c3
let c10=1/c6
do k1=1:k2
        let c8=c7*cos(2*3.14159*c6(k1)*c4)
        let c9(k1)=2*(1+2*sum(c8))
        enddo
let c11=loge(c9)
name c11 'logsdf'
let k5=8*n(c1)/(3*k2)
invcdf 0.025 k6;
chisquare k5.
invcdf 0.975 k7;
chisquare k5.
let c14=c9*k5/k6
let c15=c9*k5/k7
name c14 'UC' c15 'LC'
Let c12=c11+loge(k5/k6)
let c13=c11+loge(k5/k7)
name c12 'UCL' c13 'LCL'
set c16
k2(2)
end
name c16 'WN'
let c17= loge(c16)
Plot 'sdf'*'period' 'UC'*'period' 'LC'*'period' 'WN'*'period';
  Connect;
  Overlay.
Plot 'sdf'*c6 'UC'*c6 'LC'*c6 'WN'*c6;
  Connect;
  Overlay.
name c17 'logWN'
Plot 'logsdf'*'f' 'UCL'*'f' 'LCL'*'f' 'logWN'*'f';
  Connect;
  Overlay.
ENDMACRO
```

Figure A:1 Spectral Analysis Macro

```
                              fourier5cs.mac
Gmacro
Fourier5cs
Note Fourier Analysis of Monthly Data with Period of 12 months
Note Months in c1 (1 to 12), and Data in c2
erase c4-c18
Note: Enter Total number of data points (e.g. 120)
Note:
set c50;
file "terminal";
nobs 1.
Note
let k20=c50(1)
set c3
1:k20
end
name c3 'Index'
let k10=2*3.14159/12
let c13=sin(k10*c1)
let c14=cos(k10*c1)
let c15=sin(2*k10*c1)
let c16=cos(2*k10*c1)
let c17=sin(3*k10*c1)
let c18=cos(3*k10*c1)
let c19=sin(4*k10*c1)
let c20=cos(4*k10*c1)
let c21=sin(5*k10*c1)
let c22=cos(5*k10*c1)
name c1 'Month' c2 'Data' c13 'sin(2pt)' c14 'cos(2pt)' c15 'sin(4pt)' c16 'cos4pt'

name c17 'sin(6pt)' c18 'cos(6pt)' c19 'sin(8pt)' c20 'cos(8pt)' c21 'sin(10pt)'
name c22 'cos(10pt)'
Name c25 'COEF1' c26 'Fits' c27 'Resids'
Regress c2 10 c13-c22;
Coefficients 'COEF1';
Fits 'Fits';
Residuals 'Resids';
Gfourpack;
Brief 2.
Plot 'Data'*'Month' 'Fits'*'Month';
Symbol;
Connect;
Overlay.
NormTest 'Resids';
RJTest.
Plot 'Data'*'Fits'
Endmacro

                                Page 1
```

Figure A-2:: Fourier 5 Sine and Cosine Pair Macro